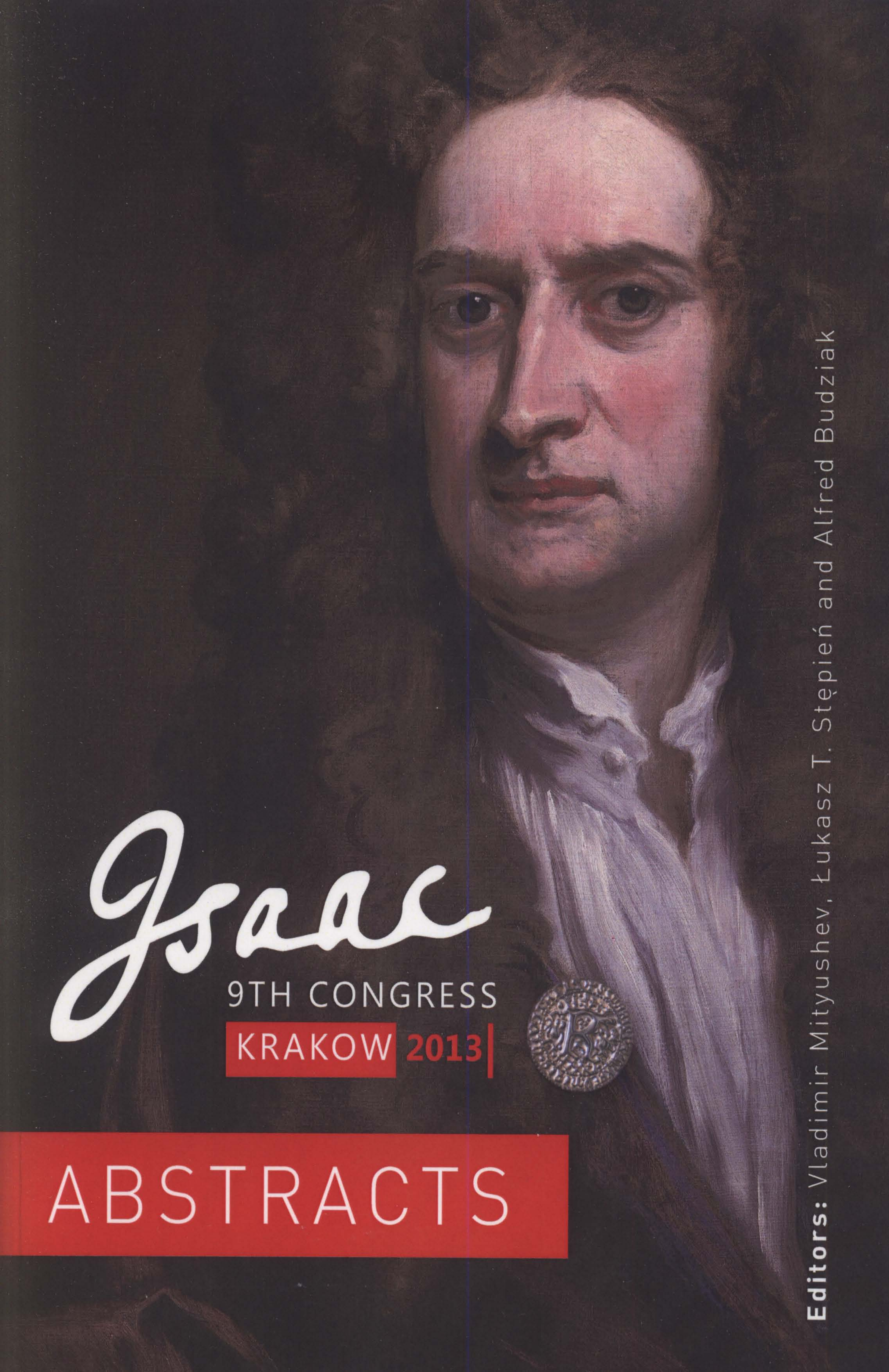

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9TH CONGRESS

KRAKOW 2013

ABSTRACTS

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Plenary Talks

■ **Pierre M. Adler** Sisyphe/UPMC Paris (France), email: `pierre.adler@upmc.fr`,

Fractured porous media

A major concern in the study of fractured media is the prediction of their properties such as the percolation of the fracture network and the macroscopic permeability from the few field quantities which are easily measurable.

Our approach is based on the systematic use of the excluded volume of fractures. When the percolation threshold of the fracture network, and the macroscopic permeability are plotted as functions of ρ', defined as the number of fractures per excluded volume, they become independent of the fracture shapes which is a decisive simplification for the applications.

ρ' can be estimated from measurements performed on intersections of fracture networks with lines, planes, and galleries. These intersections are visible on outcrops, cliffs, quarries, wells and tunnels. Some remarkable relations hold whatever the fracture shapes if they are convex.

Applications of this approach to real cases will be discussed.

During the presentation, the meshing of the fractured medium and the discretization of the equations will be detailed. This methodology can be applied to arbitrary fracture network geometries, and to arbitrary distributions of permeabilities in the porous matrix and in the fractures.

A complete presentation of our methodology and results can be found in [1].

[1] Adler, P.M., Mourzenko, V.V., Thovert, J.-F., *Fractured porous media*, Oxford University Press, Oxford (2012).

■ **Rodrigo Bañuelos, Department of Mathematics,**
Purdue University.
email: `banuelos@math.purdue.edu`

Sharp inequalities for Fourier/Lévy multipliers

We will discuss recent applications of martingale inequalities to Fourier multipliers that arise from transformations of Lévy processes via the Lévy–Khintchine formula. While these results are of interest on their own right, they are motivated by applications to problems on bounds for the Beurling-Ahlfors operator. The latter is a Calderón-Zygmund singular integral operator in the complex plane which plays a fundamental role in applications to quasi-conformal mappings. We will discuss sharp L^p and $LLogL$ (type) bounds.

Background material for this lecture can be found in the speaker's overview article [1]. The most recent results in this talk are joint work with Adam Osękowski of University of Warsaw, [2], [3].

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[2] R. Bañuelos and A. Osękowski, *On Astala's theorem for martingales and Fourier multipliers,* ArKiv:

[3] R. Bañuelos and A. Osękowski, *Sharp martingale inequalities and applications to Riesz transforms on manifolds, Lie groups and Gauss space,* **ArKiv:1305.1492v2.**

■ **First author** Richard Craster - Department of Mathematics Imperial College London, email: r.craster@imperial.ac.uk,

High frequency and dynamic homogenization

It is highly desirable to be able to create continuum equations that embed a known microstructure through effective or averaged quantities such as wavespeeds or shear moduli. The methodology for achieving this at low frequencies and for waves long relative to a microstructure is well-known and such static or quasi-static theories are well developed. However, at high frequencies the multiple scattering by the elements of the microstructure, which is now of a similar scale to the wavelength, requires a dynamic homogenization theory. Many interesting features of, say, periodic media: band gaps, localization etc occur at these higher frequencies. The materials exhibit effective anisotropy and this leads to topical effects such as cloaking/ invisibility, flat lensing, negative refraction and to inducing directional behaviour of the waves within a structure. A general theory will be described and applications to continuum, discrete and frame lattice structures will be outlined. The results and methodology are confirmed versus various illustrative exact/ numerical calculations showing that theory captures, for instance, all angle negative refraction, ultra refraction and localised defect modes.

■ **Tadeusz Czachórski** Institute of Theoretical and Applied Informatics, Polish Academy of Sciences, Gliwice, Poland, email: tadek@iitis.pl

Queueing models: transient state analysis applied to performance evaluation of computer networks

Vast volumes of data organized into packets and supervised by communications protocols are constantly flowing across telecommunications networks, usually via intermediate nodes. The waiting time at each node is unknown and depends on the current load of the network which is highly irregular. At nodes, the incoming packets are queued to be sent according to network availability. Estimation of their waiting time is important, since users need the transmission time to be as short and repeatable as possible. Reliability of transmission is equally important; when a packet queue is full, further incoming packets will not be saved. Network operators must seek a compromise: on one hand, it is important that a network be used to the best of its capacity; on the other, the more a network is being charged, the more the quality of service may drop. These issues are studied with the use of queueing theory which is present in telecommunication since the times of Erlang.

However, queueing models are usually limited to steady-state analysis, as modelling of transient states is difficult. The article discusses methods we may apply in practice to analyse in a quantitative way transient states of queues in presence of time varying input flows, namely: numerical solution of Chapman-Kolmogorov equations for continuous time Markov chains with very large state space, diffusion approximation, and fluid flow approximation. Markov models are essential for the evaluation of the performance of computer networks. However, they are not scalable: the number of states is increasing rapidly with the complexity of a modelled object. At the moment we are able to generate and solve with the use of our tool Markov chains having hundreds million of states. The method of solution is a projection method based on Krylov subspace with Arnoldi process to project the original chain onto a small Krylov subspace. In diffusion approximation a diffusion equation (second order partial differential equation) defining the position of a particle in diffuse motion is used to describe the probability distribution of a queue length. This approach is merging states of the considered queueing system and needs much less computations than the Markov models. The principles of the approximation were given in

[1] and then extended in [2] to the analysis of transient states with the use of semi-analytical, semi-numerical approach. Fluid-flow approximation is a simplified version of this method – only mean values of packet flows, queue length and service times are considered. Differential equations (first order linear ordinary differential equations) inolved here are simpler, and the computations can be completed in a reasonable time even for very large network topologies. It is also easier to model mechanisms used to control queues in nodes.

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■ **First author** Erol Gelenbe - Imperial College London, email: e.gelenbe@imperial.ac.uk,

The Time and Energy Needed for Search in Very Large Spaces

Many applications in biology, science and engineering involve search in random media, and optimisation often involves stochastic search. In many cases the search space is for all practical purposes infinite. Examples include the motion of particles towards an opposite charged site in a random field, molecules moving in a medium in search of other molecules that they may dock with, search for specific data in very large data bases, packets in ad-hoc networks, or robots searching for a concealed object. In many cases one uses multiple searchers. In many natural examples the search may take place as a group effort where at least k members of the swarm out of N must be successful. In all such applications, the length of time it takes for the search to conclude successfully is of interest, and it is important to know how much energy is consumed. Thus this presentation will summarise some of our recent work [12, 14] in this area. The analysis will include the probability of loss or destruction of the searchers, a limited life-time or time-out for each of the searchers, and will allow for sending out (after some time) of a successor to a searcher that is lost, destroyed or that has been eliminated because of the time-out.

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■ Tokio Matsuyama Tokio Matsuyama - Chuo University, Tokyo, email: tokio@math.chuo-u.ac.jp

Global well-posedness of the Kirchhoff equation and systems

This talk is devoted to review the global well-posedness for the Kirchhoff equation. The classical results on Kirchhoff equation with large data will be introduced (see [1, 2, 6, 10, 11]), and then, some results on Kirchhoff systems with small data are reviewed, which are joint works with Michael Ruzhansky. The results cover the classical Kirchhoff equation that are discussed in [3, 12] etc. The new approach to the construction of solutions is based on the asymptotic integrations for strictly hyperbolic systems with time-dependent coefficients. These integrations play an important role to setting the subsequent fixed point argument. The existence of solutions for less regular data is discussed, and several examples and applications are presented.

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■ **First author** Wieslaw L. Nowinski - Biomedical Imaging Lab, Agency for Science Technology and Research, Singapore, email: Wieslaw@sbic.a-star.edu.sg

Brain atlasing: methods, tools and applications

We recently witness unprecedented big brain projects, including The BRAIN Initiative and The Human Brain Project. Our contribution to brain-related efforts is to create brain atlases and develop atlas-based applications. The latest generation of our atlases has been constructed from 3/7 Tesla scans. The brain model has been parcellated into about 3,000 3D components. The atlas design principles, methods, tools, content, features, and validation were described earlier [1]-[6]. Several techniques have been applied to build the atlas, including image processing, intra-subject registration, segmentation, geometric modeling, surface and tubular editing, classification, and atlas-to-patient registration. We have developed a number of editors for creation of cerebral objects, and their editing and enhancement [2], [11]. This brain atlas is a foundation for development of education, research and clinical applications. The Human Brain in 1969 Pieces [7] is a neuroanatomical reference and an education tool. Its iPad version is also developed [8]. An atlas-based system for functional neurosurgery planning, aiming at reducing hemorrhage induced by deep brain stimulation, is featured in [6]. A recent application is the atlas of neurological disorders correlating localized brain pathology with both the resulting disorder and the surrounding neuroanatomy [10]-[11].

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■ **First author** Lassi Päivärinta - Department of Mathematics and Statistics/Rolf Nevanlinna Institute, University of Helsinki FINLAND, email: `lassi.paivarinta@helsinki.fi`,

Corner Scattering

The talk is connected to scattering theory and especially to inverse scattering problem. In the inverse scattering one wants to define an unknown structure from far field measurements i.e.. from the scattering amplitude. A related problem is wether structures always produce a scattering wave. The question is related to a recently extensively studied Interior Transmission Problem and to Interior Transmission Eigenvalues. In the radially symmetric case it is known that this is not the case i.e.. there are wave numbers, called non-scattering wave numbers, and incident fields that do not cause any response to infinity. In the talk we show that in the case where the structure contains a corner a right angle every wave and every wave number scatters.

This is a joint research with Eemeli Blåsten and John Sylvester.

■ **Grigory Panasenko** Inst. Camille Jordan, Univ. Saint-Etienne, France,
email: grigory.panasenko@univ-st-etienne.fr

Junction of models of different dimension: wall-fluid interaction and bio-mathematical applications

Junction of models of different dimension is an important tool of the multiscale modeling of the blood flow [1]. We discuss this topic in the context of the partial differential equations (PDEs) set in thin structures. These domains are some finite unions of thin rectangles (in 2D settings) or cylinders (in 3D settings) depending on small parameter epsilon ¡¡ 1 that is, the ratio of the thickness of the rectangle (cylinder) to its length. Such a domain models the system of the blood vessels where blood flow occurs. An asymptotic analysis of the solutions of these PDEs is applied for justification of the method of asymptotic partial decomposition of domain (MAPDD) introduced in [2] and developed in [3]. Remind that this method reduces the 2D or 3D model to some model of hybrid dimension (2-1 or 3-1) conserving the dimension on a small part where the behaviour of solution is singular, and reducing the dimension in the main part of the domain. This approach corresponds to a special dimension reduction procedure with some local zooms and is applied further to the flows in thin domains with an elastic rigid wall.

The idea is to keep the 3D (or 2D) description of the flow-wall interaction in some neighbourhood of the bifurcations of vessels and to reduce the dimension in the cylinders (rectangles) at some distance from the bifurcations. First, the question how to replace the wall of the cylinder (or rectangle) by some special boundary condition is discussed. The rigidity (and eventually the density) of the wall material are great parameters related to some negative power of the ratio of the thickness of the wall to the thickness of the channel. The second question is how to replace the flow in a thin channel with these special elastic wall boundary conditions by some analogue of the Poiseuille flow and to make a junction of this flow to the real flow in the neighbourhood of the bifurcations. These questions are studied via construction of asymptotic expansions of flows in a channel with a rigid elastic wall bifurcations (see, for example, [4],[5]). Finally this approach gives an algorithm of junction of fluid-structure interaction models of different dimension.

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■ **Rusakov A. A.** professor, Moscow Lomonosov State University, vmkafedra@yandex.ru

S. M. Nikolsky. Life as an example

Sergey was born in the village of Plant Talitsa Perm province (now Yekaterinburg regional center Talitsa area) April 30, 1905. His father, Michael D. Nikolsky entered the body of forestersRussian forest range. By bureaucratic report card forestercourt counselor in 1906 by M. D. Nikolsky gets promoted, he was appointed forester Schebro-Olshansky forest Suvolkskoy province on the border with Prussia (now Poland), where Sergey spent his childhood.

After graduating from the Faculty of Physics and Mathematics Ekaterinoslavskogo Institute of Education, since 1930 Sergey Nikolsky Assistant University. As the best lecturer, since 1932 head of the department of mathematics Transport Institute, also works in the mining, pharmaceutical institutes of Dnipropetrovsk.

The scientific work of Sergey Nikolsky involved Academician Andrei Nikolaevich Kolmogorov, who came from Moscow with Academician P.S. Aleksandrov and Ivan Petrovsky to give lectures. In 19341935, was in graduate school at Moscow State University. M.V. Lomonosova, defended his thesis on "Linear equations in Banach spaces."

In 1940, S. M. Nikolsky entered the doctoral program of the Mathematical Institute. VA USSR Academy of Sciences and in early 1942, successfully defended his doctoral thesis on the theory of approximation of functions by polynomials. Was left a senior fellow at the Mathematical Institute. In 1947 he became a professor at the Department of Mathematics of the same institute, and from 1950 to 1954. was the head of this department. From 1953 to 1961. was deputy director from 1961 to 1989. - Head of department of the theory of functions. In 1968. S. M. Nikolsky was elected a corresponding member of the Academy of Sciences of the USSR, and in 1972a full member.

In 2006, the series "Monuments of national science XX century" of the Russian Academy of Sciences published a three-volume collected works of S. M. Nikolsky. The best results obtained by S. M. Nikolsky in fundamental mathematics, collected and published in his three monographs: "Approximation Theory" (2006), "Function Spaces" (2007) and the third is"The equations in functional spaces" (2009).

S. M. Nikolsky has trained about fifty candidates of physical and mathematical sciences, fifteen of his studentsthe doctor of physical and mathematical sciences. Among his students are well-known scientists as corresponding member of the Russian Academy of Sciences, member of the European Academy of Sciences O. V. Besov, Corresponding Member of Russian Academy of Sciences dents, Member of the European Academy of Sciences, L. D. Kudryavtsev, a seminal work of more than justified the bold decision to teachers, appointed him to the age of thirty head of the Department of Mathematics of the leading university of the country - MFTI, works L. D. Kudryavtsevmost the best proof of the power of the scientific school of S. M. Nikolsky, Professor A. F. Timman, corresponding member of the Academy of Sciences of the USSR V. K. Dzyadyk, National Academy of Sciences of Ukraine, N. P. Korneichuk, corresponding member of the National Academy of Sciences of Ukraine V. P. Motorny, corresponding member of the Academy of Sciences of the Kazakh SSR T. I. Omanov, Professor S. V. Uspensky, Professor V. I. Burenkov, emeritus professor of the Moscow State University, M. K. Potapov, etc.

■ **Stefan Samko** Stefan Samko - Universidade do Algarve, Faro, Portugal, email: `ssamko@ualg.pt`,

Variable exponent Morrey and Herz spaces

We present several recent results in the rapidly developing area of of Analysis known as Variable Exponent Analysis but start with some new results for constant exponents. We concentrate on the following topics:

I. Close embedding of Morrey spaces between vanishing Stummel spaces.

II. Maximal and potential operators in local variable exponent Morrey spaces over unbounded sets.

III. Sublinear operators of singular type in Variable exponent Herz spaces .

■ **First author** Baoxiang Wang - Peking University, Beijing, China, email: `wbx@pku.edu.cn`

Frequency-Uniform Decomposition Techniques for a Class of Nonlinear Partial Differential Equations

In this talk, we survey some recent results on the Cauchy problem for a class of nonlinear partial differential equations, including Navier-Stokes equations, (derivative) nonlinear Schrödinger equations by using the frequency-uniform decomposition method and the Gabor frames.

■ **Henryk Woźniakowski**

Henryk Woźniakowski - Columbia University and University of Warsaw, email: `henryk@cs.columbia.edu`,

Tractability of Multivariate Problems

Multivariate problems are defined on spaces of functions of d variables, where d may be huge. Such problems occur in numerous applications and can almost never be solved analytically. Usually we want to solve them to within ε and we may measure the error in the different settings including the worst case, average case, or randomized setting. For simplicity we concentrate on the worst case setting.

Usually such problems suffer the *curse of dimensionality*, i.e., the number of computational operations is exponential in d. This is the case for most multivariate problems defined on standard spaces of functions for which all variables and groups of variables are equally important. The essence of *tractability* study is to identify classes of functions for which multivariate problems do *not* require the exponential number of operations in d and ε^{-1}. It turns out that tractability usually holds for multivariate problems defined on *weighted* spaces for which the successive variables or groups of variables are monitored by suitable weights.

The current state of tractability studies for multivariate problems can be found in [1].

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■ **M.Zhuravkov** Belarusian State University, Minsk, 4 Nezavisimost av. email: `Zhuravkov@bsu.by`, **N.Romanova** Belarusian State University, Minsk,4 Nezavisimost av. email: `RomanovaNS@bsu.by`

Fractional models for determination of physical and mechanical properties biological tissue

Solving inverse problem is an important direction in an investigation of physical and mechanical characteristics of various heterogeneous materials. Recently, such approaches are being actively implemented to determine the properties of nanomaterials due to the increased use of the techniques that are based on scanning force microscopes (SFM).

In our paper the problem of the restoring biotissue properties is considered as a processing of the experimental data obtained in AFM indentation of living cells. Here AFM (Atomic Force Microscopy) or SFM (Scanning Force Microscopy) is a very high-resolution type of scanning probe microscopy that provides a 3D profile of the surface on a nanoscale [1]. A consideration of the physical mechanical properties of living cells is very important for an investigation of cell processes when influence some pathogenic factors (viruses and bacteria) or pharmacological preparations [2], [3].

Mechanic mathematical model problem is a process modelling of the occurrence and movement of surface and shear waves in a viscoelastic half-space [4]. The obtained results show that fractional Voigt model (time-fractional order derivative) gives a more adequate results than standard integer order Voigt model or standard linear solid model. Same number of independent parameters is used in all considered models.

Fractional viscoelastic models, the accuracy of which can be increased through the introduction of the additional fractional elements, can be widely used for early medical diagnosis by biological tissue testing that is shown in our paper.

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Session Talks

■ **Luís Daniel Abreu** -ARI, Austrian Academy of Sciences, email: daniel@mat.uc.pt
Monica Dörfler - NuHAG, University of Viena, email: monika.doerfler@univie.ac.at

Inverse problems for Toeplitz and time–frequency localization operators

A new type of inverse problems for Toeplitz and time–frequency localization operators will be presented.

A result of time-frequency analysis, obtained by Daubechies in 1988, states that the eigenfunctions of a time-frequency localization operator with Gaussian window (related to Fock spaces) such that the localization domain is a disc are the Hermite functions. In [1] we have have proved the following converse of Daubechies result:

Theorem: *If one of the eigenfunctions of a time-frequency localization operator with Gaussian window is a Hermite function, then its localization domain, when simply connected, is a disc.*

There is an analogue problem for wavelet localization in hyperbolic geometry, which states that the eigenfunctions of a time-frequency localization operator with a Cauchy window (related to Bergman spaces) such that the localization domain is a disc are the Fourier transform of Laguerre functions., providing the inverse problem analogue of the direct problem studied by Daubechies and Paul.

Theorem: *If one of the eigenfunctions of the wavelet localization operator with a Cauchy window is a Fourier transform of a Laguerre function, then Δ must be a pseudohyperbolic disc centered at i.*

We will also present a no-go example concerning generalizations to more general symbols, due to Gröchenig, as well as a number of natural questions that arise naturally when one looks at this problem in other contexts, like Modulation spaces and spaces of polyanalytic functions.

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On the boundary behavior of the generalized ring Q-homeomorphisms in metric spaces

Let us consider the following system of Borel measures which are associated with continua in metric spaces (X, d). A measure $m_\gamma^{(k)}$ is associated with continuum γ in (X, d) is defined for every Borel set B in (X, d) as Hausdorff measure H^k of $B \cap \gamma$ at fixed $k > 0$. For every continuum $\gamma \in \Gamma$, set $m_\gamma := m_\gamma^{(1)}$. Now, let (X, d, μ) be a metric space with a Borel measure μ. μ-measurable functions $\rho : X \to [0, \infty]$, satisfying $\int\limits_X \rho\, dm_\gamma \geq 1$, for all curves $\gamma \in \Gamma$ are called **admissible functions** for Γ, abbr. $\rho \in adm\Gamma$. p**-modulus**, $p \in (0, \infty)$, of Γ is the quantity $M_p(\Gamma) = \inf\limits_{\rho \in adm\,\Gamma} \int\limits_X \rho^p(x)\, d\mu(x)$ where the infimum is taken over all $\rho \in adm\Gamma$. $M_p(\Gamma) = +\infty$ if $\Gamma = \emptyset$. Later $\Gamma(A, B; C)$ denotes the family of all continua $\gamma \in \Gamma$ connecting A and B in C, i.e., such that $\gamma \cap A \neq \emptyset$, $\gamma \cap B \neq \emptyset$ and $\gamma \setminus \{A \cup B\} \subseteq C$. A **generalized domain** in topological space T is an open set D whose each pair of points can be immersed in a continuum γ in D.

Let D and D' be generalized domains in (X, d, μ) and (X', d', μ'), respectively, $Q : X \to (0, \infty)$ be a μ-measurable function and $p \in (0, \infty)$. We say that homeomorphism $f : D \to D'$ is a **generalized ring Q-homeomorphism at a point** $x_0 \in \overline{D}$ with respect to p-module, if $M_p(\Gamma(f(C_0),\, f(C_1);\, D')) \leq \int\limits_{A \cap D} Q(x) \cdot \eta^p(d(x, x_0))\, d\mu(x)$ for every ring $A = A(x_0, r_1, r_2)$ $0 < r_1 < r_2 < \infty$, for any two continua $C_0 \subset \overline{B(x_0,\, r_1)} \cap D$ and $C_1 \subset D \setminus B(x_0, r_2)$ and for every Borel function $\eta : (r_1, r_2) \to [0, \infty]$, such that $\int\limits_{r_1}^{r_2} \eta(r)\, dr \geq 1$. A homeomorphism $f : D \to D'$ is called a generalized ring Q-homeomorphism if f is a generalized ring Q-homeomorphism at every point $x_0 \in \overline{D}$. A generalized domain D in (X, d, μ) is called a **generalized quasiextremal distance (QED) domain** with respect to p-module, $p \in (0, \infty)$, if $M_p(\Gamma(E, F; X)) \leq K\, M_p(\Gamma(E, F; D))$ for some $K \in [1, \infty)$ and for every continua E and F in D, cf. [1]. We also say that a space (X, d, μ) is **weakly flat at a point** $x_0 \in X$ with respect to p-module, $p \in (0, \infty)$, if, for every neighborhood U of the point x_0 and every number $N > 0$, there is neighborhood $V \subseteq U$ of x_0, such that $M_p(\Gamma(E, F; X)) \geq N$ for any continua E and F in X intersecting ∂V and ∂U, cf. [2].

Theorem. Let f be a generalized ring Q-homeomorphism with respect to p-module, $p \in (0, \infty)$, between QED generalized domains D and D' in weekly flat spaces X and X', respectively, and let $\overline{D}$ be compact. If $Q \in L^1_\mu(D)$, then the inverse homeomorphism $g = f^{-1}$ admits a continuous extension $\overline{g} : \overline{D'} \to \overline{D}$.

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Compactness via the Berezin transform of Radial Operators on the generalized Fock spaces

We investigate the connection between compactness of radial operators on the generalized Fock space and the behavior at infinity of the corresponding Berezin transform and give a condition under which compactness of an operator is equivalent to the vanishing of the Berezin transform at infinity. Our result is based on a certain o-Tauberian theorem due to Kranz and Stadtmüller. We also show that the eigenvalues of radial operators depend only on the length of the multi-index.

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Elementary Approach to Fractional Powers of Strongly Elliptic Operator in Lipschitz Domains

Let L be a second-order matrix partial differential strongly elliptic operator in a bounded Lipschitz domain $\Omega \subset R^n$, $n > 1$. Extensive literature is devoted to the investigation of domains of fractional powers of this operator with Dirichlet, Neumann, or mixed homogeneous boundary conditions, including the solution of the well-known Kato square root problem. The higher-order matrix operators were also considered.

We propose a new abstract approach to these problems. It allows us to obtain the main, from our point of view, results in a unique and simple way, as well as new similar results, in particular, for classical operators on a Lipschitz boundary.

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Jackson and smoothness theorems for trigonometric approximation in some Banach Function Spaces

A. Let $\mathbb{T}_n$ be the class of trigonometrical polynomials of degree at most n and let X be a Banach function space (see e.g., [1]) on $T := [0, 2\pi]$ such that (i) $\mathbb{T}_n$ be a dense subset of X, (ii) the Steklov type operator $f(\cdot) \mapsto \sigma_h f(\cdot) := (2h)^{-1} \int_{x-h}^{x+h} f(t)dt$ be bounded on X. We define the classical de la Valleé Poussin mean $V_n f(\cdot) := (n+1)^{-1} \sum_{i=n}^{2n} S_i(\cdot, f)$ of $f \in X \subset L^1$. $\Omega_r(f, \delta)_X := \sup_{0 \le h \le \delta} \|(I - \sigma_h)^r f\|_X$. We proved.

* Let X be as described in A. If the operator $f \mapsto V_n$ be bounded on X and $E_n(f)_X := \inf_{T \in \mathbb{T}_n} \|f - T\|_X \lesssim n^{-2} \|f''\|_X$ for any $f \in X'' := \{f \in X : f'' \in X\}$, then we have the following Jackson-Stechkin type estimate

$$E_n(f)_X \lesssim \Omega_r(f, 1/n)_X$$

for any $r = 1, 2, 3, ...$ and $n = 1, 2, 3, ...$

* Let X be as described in A. If the inequality $\|T_n'\|_X \lesssim n \|T_n'\|_X$ for any $T_n \in \mathbb{T}_n$ holds, then we have the following converse estimate

$$\Omega_r \left(f, \frac{1}{n} \right)_X \lesssim \frac{1}{n^{2r}} \sum_{j=0}^{n} (j+1)^{2r-1} E_j(f)_X$$

for any $r = 1, 2, 3, ...$ and $n = 1, 2, 3, ...$

Some examples for X:

1. Classical Lebesgue spaces L^p with $1 \le p \le \infty$.
2. Weighted Lebesgue spaces L^p_ω, $1 < p < \infty$ with weights ω satisfying the Muckenhoup's condition A_p.
3. Variable exponent Lebesgue spaces $L^{p(\cdot)}$ with $p(\cdot)$ satisfying $1 < essinf_T p(x)$, $esssup_T p(x) < \infty$ and the Dini-Lipschitz condition of order ≥ 1. **4.** Weighted variable exponent Lebesgue spaces $L^{p(\cdot)}_\omega$ with the conditions $1 < essinf_T p(x)$, $esssup_T p(x) < \infty$, $1/p$ Log-Hölder continuous, $\omega^{p_0} \in A_{(p(\cdot)/p_0)'}$ for some $p_0 \in (1, essinf_T p(x))$. **5.** Orlicz spaces $\varphi(L)$ with quasiconvex φ^α for some $\alpha \in (0, 1)$ and $\varphi \in \Delta_2$. **6.** Weighted Orlicz spaces $\varphi_\omega(L)$ with quasiconvex φ^α for some $\alpha \in (0, 1)$ and $\varphi \in \Delta_2$ and $\omega \in A_{p(\varphi)}$. **7.** Rearrangement invariant Banach function spaces having Absolutely Continuous Norms with non-trivial Boyd indices.

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About one system of partial differential equation with singular coefficients

Suppose $0 < \varphi_0 \leq 2\pi$, $0 < \varphi_1 < \varphi_2 < \varphi_0$, $S[0, \varphi_0]$ be the set of measurable, essentially bounded functions $f(r, \varphi)$ in $[0, \varphi_0]$. Let us consider in $G = \{z = re^{i\varphi} : 0 \leq r < \infty, 0 \leq \varphi \leq \varphi_0\}$ equation

$$2\overline{z}a_1(\varphi)\partial_{\overline{z}}w + 2za_2(\varphi)\partial_z w + \frac{r^\alpha b_1(\varphi)w}{|f_1(x,y)|^\alpha} + \frac{r^\alpha b_2(\varphi)\overline{w}}{|f_2(x,y)|^\alpha} = \frac{f_4(\varphi)r^{\nu+\alpha}}{|f_3(x,y)|^\alpha},$$

where $a_1(\varphi), a_1(\varphi) \in C[0, \varphi_0]$; $a_1(\varphi) \neq a_2(\varphi)$ for all $\varphi \in [0, \varphi_0]$; $0 < \alpha < 1$, $\nu > 0$ are real numbers.

Suppose that the functions $f_k(x, y), k = 1, 2, 3$ be first order homogeneous functions i.e. $f_k(nx, ny) = nf_k(x, y)$ for any real number n and $\frac{f_4(\varphi)}{|f_3(\cos\varphi,\sin\varphi)|^\alpha}, \frac{b_1(\varphi)}{|f_1(\cos\varphi,\sin\varphi)|^\alpha} \in L_1[0, \varphi_0]$, $\frac{b_2(\varphi)}{|f_2(\cos\varphi,\sin\varphi)|^\alpha} \in S[0, \varphi_0]$.

Let us $p > 1$, if $\nu \geq 1$ and $1 < p < \frac{1}{1-\nu}$, if $\nu < 1$. In this work the solutions from class $W_p^1(G) \bigcap C(G)$ are found.

Here $W_p^1(G)$ is a Sobolev space. By $f_1(x, y) = y - k_1x$, $f_2(x, y) \equiv 1$, $f_3(x, y) = y - k_2x$; $k_1, k_2 \in \mathbb{R}$ the considered equation is studied in work [1]. By $\alpha = 0$, $a_2(\varphi) \equiv 0$ equation is studied in [2].

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On Some Boundary Value Problems for linear partial differential equations of second order in Clifford Analysis

We give the solutions of Dirichlet and Neumann problems for the Poisson equation in the unit ball using Cauchy-Pompeiu type representations in the complex Clifford algebra $\mathbb{C}_m$ for $m \geq 3$. By introducing classes of integral operators together with some of their properties, we study the Dirichlet and Neumann problems for linear partial differential equations of second order in Clifford analysis.

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Differential algebra of biquaternions.
Twistors equation and its generalized solutions

On base of differential algebra of biquaternions and theories of generalized function the biquaternionic wave (*biwave*) equation of general type is considered under vector representation of its structural coefficient. Its generalized decisions, describing nonstationary, harmonic and static elementary twistors and twistor fields, are built.

With use of differential operators - mutual complex gradients (*bigradients*):

$$\nabla^{\pm}\mathbf{B} = (\partial_\tau \pm i\nabla) \circ (b + B) = \partial_\tau b \mp i \,\mathrm{div}\, B + \partial_\tau B \pm i \mathrm{grad} B \pm i \,\mathrm{rot}\, B,$$

which generalize the notion of gradient on biquaternionic functions on Minkowski space (M), we consider differential biquaternionic biwave equation:

$$\nabla^{\pm}\mathbf{B} + F \circ \mathbf{B} = \mathbf{G}(\tau, x), \quad \mathbf{B} \in \mathsf{B}'(\mathsf{M}), \tag{1}$$

structural coefficient F is 3D-vector with complex components. If to write this equation in matrix (tensor) form, it pertains to class of the Young-Mills equations [1], which are used in theoretical physics for mathematical description of elementary particles.

Using the property of mutual bigradients composition [2] the generalized decisions of Eq.(1) are built, including steady-state case and the case of stationary vibrations. In particular, for homogeneous equation (when $\mathbf{G} = 0$) its solutions, named *twistors*, are obtained as the biquaternionic convolution

$$\mathbf{Tw} = (\partial_\tau \psi \mp \mathrm{grad}\psi - F\psi) * \mathbf{C}(\tau, x), \quad \mathbf{C} \in \mathsf{B}'(\mathsf{M}), \tag{2}$$

where *scalar potentials* ψ are solutions of the following equation:

$$\Box\psi + (F, F)\psi - 2i(F, \nabla\psi) = 0, \tag{3}$$

which also are constructed.

The elementary twistors are constructed and their wave properties are investigated.

Used for building of these decisions methods are in detail described by author in [2] for biwave equations only with scalar structural coefficient.

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SHOCK THERMOELASTIC WAVES AS GENERALIZED SOLUTIONS OF THERMOELASTICY EQUATIONS

Investigation of wave processes in media belongs to actual problems of mathematical physics. It's connected with solving Boundary Value Problems (BVPs) for hyperbolic equations systems and systems of mixed type. Being accompanied with shock waves, such processes are described by functions, which are not differentiable on its fronts. That essentially complicates using the classical methods of BVPs theory for elliptical systems for their studying.

In this paper we consider dynamic processes in thermoelastic media taking into account shock thermoelastic waves. By use of generalized functions theory the conditions on jumps of stresses σ_{ij}, velocities $u_{,t}$, gradients of temperature θ and energy density $W(u,\theta)$ on their fronts are received:

$$[u]_{F_t} = 0, \quad [\theta]_{F_t} = 0, \quad n_j\,[\sigma_{ij}]_{F_t} = -\rho c\,[u_i,_t]_{F_t}\,, \quad n_j\,[\theta,_j]_{F_t} = \eta n_j\,[u_j,_t]_{F_t}$$

$$[W(u,\theta)]_F = -c^{-1}\left[n_i(\dot{u}_j,\sigma_{ij}) + \gamma\eta^{-1}\theta\frac{\partial\theta}{\partial n}\right]_F$$

here ρ, c are density and velocity of wave fronts propagation,n is normal vector on wave front.

The law of energy conservation for the thermoelastic medium has been proved subject to shock waves. The statements of four non-stationary BVPs of coupled thermoelasticity are given, for which uniqueness of decisions are proved taking into account shock thermoelastic waves.

On the basis of Generalized Functions Method the singular Boundary Integral Equations Method was developed by authors for solving the non-stationary BVPs of thermoelastodynamics [1,2], which can be used also for processes with shock thermoelastic waves.

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Riesz and Wolff potentials in variable exponent Lebesgue spaces

We study mapping properties of Riesz and Wolff potentials in Lebesgue spaces with variable exponent. Our approach includes the limiting case corresponding to potentials of L^1 functions. This case is particularly interesting in the study of some nonlinear PDEs.

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Some classes of slice hyperholomorphic functions and associated reproducing kernel Pontryagin spaces

In the 1970's Krein and Langer introduced and studied a number of classes of functions meromorphic in the open unit disk or in half-plane with an associated kernel having a finite number of negative squares (the latter is a generalization of the notion of positive definite functions). These functions played a key role in extending Schur analysis to the non-positive case, that is when Hilbert spaces are replaced by Pontryagin spaces. In the talk we discuss the counterparts of these classes in the slice hyperholomorphic setting. We discuss the counterpart of the characteristic operator functions and give relationships with the Kalman-Yakubovich-Popov lemma.

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Simulation of oil spills spreading on water surface using neural network methods

Spreading oil in the sea is a complicated process. To describe this process it is required to take into consideration a number of various factors. The main factors are the volume of spilled oil, oil density, viscosity index, sulfur content and content of other impurities in oil, water temperature, ambient temperature and chemical properties of water. All these factors are required for calculating oil slick height and spill area. Let us denote the spilled oil volume by X_1 and oil density by X_2 and do this for all factors respectively. The last factor, which is a chemical composition of water, is denoted by X_7. Output factors for our model are oil slick height denoted by Y_1, and area of a spill denoted by Y_2. The same parametrs use in [1]. We will use network of neural methods for simulation.

Let us take the matrix of a training set to get the global minimum of the network. Let us have a network of neural converting $F : X \to Y$ vectors X from input attribute space to output vector space Y. Network state space is denoted by W.

Let us assume that there is the training set (X^α, Y^α), $\alpha = 1 \ldots p$.

Let us consider the total error E under the network state W

$$E = E(W) = \sum_\alpha \|F(X^\alpha; W) - Y^\alpha\|$$

Let us emphasize two more properties of network state error. First error E=E (W) is a function of W state determined in state space. By convention it has nonnegative values. Second, under some trained state where the network does not make errors on the training set, this function takes zero value. Practically the Zero value is unreachable and thus we train the network to set error E. Consequently trained states are the minimum points of the entered function $E(W)$. There can be a large number of such minimum points and testing set allows selecting required global minimum.

Then using a newly created network and such input parameters as wind speed, flow speed, and area of spilled oil, calculated by means the previous network, we can calculate such input parameters as drift direction and shifting of oil slick gravity center. Thus, by dividing the task into 2 parts we can calculate the area and height of oil slick using the first network and then calculate gravity center and oil slick direction using the second network.

Having the volume, direction and deviation distance we can identify oil slick state. The testing data, in particular, have been checked by the application program Neural Planner.

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Boundary value problem for the radiation transfer equation with reflection and refraction conditions

The boundary value problems for the equation of radiation transfer with the reflection and refraction conditions at the interface are still studied unsatisfactory, in spite of the importance of such problems in applications. Such problems with the reflection and refraction conditions in accordance with the Fresnel laws were first studied in the papers [1], [2].

We study the boundary value problem for the equation of radiation transfer

$$\omega \cdot \nabla I + (\varkappa + s)I = s\mathcal{S}(I) + \varkappa k^2 F, \quad (\omega, x) \in \Omega \times G$$

in the system $G = \cup_{j=1}^{m} G_j$ of semitransparent bodies $G_j \in \mathbb{R}^3$ separated by the vacuum under the condition of mirror reflection and refraction according to the Fresnel laws at the interface between media. Here $\omega \cdot \nabla I$ denotes the derivative of a function I along the direction ω and $\mathcal{S}$ denotes the scattering operator. We assume that the domains G_i and G_j are pairwise disjoint, whereas the boundaries of G_i and G_j can intersect for some $i \neq j$. The sought function $I(\omega, x)$ is interpreted as the radiation intensity at a point $x \in G$ when the radiation propagates along the direction $\omega \in \Omega$, where Ω is the unit sphere in $\mathbb{R}^3$ (the sphere of directions). Each G_j is occupied by a semitransparent medium with constant absorption $\varkappa_j > 0$, and scattering $s_j \geq 0$ coefficients and the refraction exponent $k_j > 1$.

We established the existence and uniqueness of a solution $I \in \mathcal{W}^p(D) = \{I \in L^p(D) \mid \omega \cdot \nabla I \in L^p(D)\}$, $1 \leq p \leq \infty$ [3]. We also obtained a priori estimates for the solution.

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Harmonic Scalar Waves in Waveguides with Losses

When using Fourier methods to solve scalar wave scattering problems in waveguides with losses [1], the set of eigenfunctions lack some of the important properties that makes the lossless case simple and straightforward. For example, in a two-dimensional horizontal waveguide with a hard lower bound at $y = 0$ and a non-hard upper bound at $y = 1$, we have for each x eigenfunctions $\cos(\lambda_n(x)y)$, where the complex numbers $\lambda_n(x)$ are solutions to the equation

$$\lambda \sin \lambda = \mathrm{i}k\beta(x) \cos \lambda. \tag{1}$$

In (1), k is the wave number and $\beta(x)$ is the boundary admittance at x. The solutions are, if $\beta(x)$ is not pure imaginary, non-real, and the standard theory regarding orthogonality and completeness for the corresponding eigenfunctions are not applicable.

We will investigate the location of roots to equation (1) and show that the corresponding eigenfunctions form a basis for the Hilbert space $L^2(0,1)$.

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Calabi-Yau threefolds, stable bundles and applications in string theory

Three approaches to construct holomorphic vector bundles, with structure group the complexification $G_{\mathbb{C}}$ of a compact Lie group G, on elliptically fibered Calabi-Yau threefolds have been introduced in [1]. The parabolic bundle approach applies for any simple G. One considers deformations of certain minimally unstable G-bundles corresponding to special maximal parabolic subgroups of G. The spectral cover approach applies for $SU(n)$ and $Sp(n)$ bundles and can be essentially understood as a relative Fourier-Mukai transformation which also describes coherent sheaves or cohomological complexes of such sheaves. Finally, the del Pezzo surface approach applies for E_6, E_7 and E_8 bundles and uses the relation between subgroups of G and singularities of del Pezzo surfaces.

In this talk, we briefly review these approaches and discuss then the general question how to give sufficient conditions for the existence of stable bundles with prescribed Chern classes on Calabi-Yau threefolds. We describe a conjecture which has been put forward by Douglas, Reinbacher and Yau giving such conditions and prove a weak form of the conjecture following [3]-[5].

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H–distributions: an extension of H–measures

H–measures were introduced by Luc Tartar (1990), and independently by Patrick Gérard (under the name of microlocal defect measures, 1991), and proved to be a powerful tool for treating weakly converging sequences of solutions to partial differential equations. Recently, a number of variants has been introduced (see the introduction in [3]).

However, H–measures are essentially adapted to the L^2 framework, and we proposed [1] an extension which can also treat the $\mathrm{L}^p - \mathrm{L}^q$ pairing. The proof of their existence is based on the classical multiplier theorems.

The general framework for construction of variant H–measures and analogous objects will be discussed, together with some recent extensions and applications [2, 4, 5].

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Longitudinal impact through a spring: Optimization and inverse problems

A longitudinal elastic compressed stress wave impulse is defined as one-dimensional wave propagation resulting in a long elastic rod from a rapid loading process (for instance, short-time impact with a rigid hummer). St. Venant's problem of an elastic rod with a fixed end impacted by a moving mass at the other end was studied in a number of papers [1, 2, 3, 4]. It was shown that the impactor rebound is not possible before the impact wave returns after reflection at the rod's fixed end. Thus, in the case of impact of a rigid mass against a semi-infinite elastic rod, there will be no rebound at all (see, [5], Section 2.4.2). The problem of a semi-infinite elastic rod struck by a moving mass through a linear spring was considered in [6], where it was shown that a finite-time impact occurs for a relatively stiff spring.

In the present contribution, the optimization problem resulting in generating compressed pulses of minimum duration is addressed for a semi-infinite elastic rod struck by a rigid mass through a linear spring. The optimal value for the spring stiffness is presented in a closed form. Inverse problems of recovery of the impact system parameters from the parameters of the strain wave impulse are also considered.

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Hartog's Type Extension Theorem in Some Piecewise Constant Structure Relations For Metamonogenic Functions of First Order

A metamonogenic function of first order or simply a metamonogenic function is a function u which satisfies the differential equation $(D-\lambda)u=0$, where D is the Cauchy-Riemann operator and $\lambda\in\mathbb{R}$. Using this definition we can say that a multi-metamonogenic function u is separately metamonogenic in all variables x^j , $j=1,\dots n$ where $n\geq 2$ and $x^j=(x_1^{(j)},\dots x_{m_j}^{(j)})$ runs in the Euclidean space $\mathbb{R}^{m_j}$, that is $(D_j-\lambda)u=0$, for each $j=1,\dots,n$, with D_j being the Cauchy-Riemann operator corresponding to the space $\mathbb{R}^{m_j}$. The main purpose of this work is to study a Hartog's type extension theorem for metamonogenic and multi-metamonogenic functions, with a Clifford type algebraic structure, called, some Piecewise Constant Structure Relations (PCSR).

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The Cauchy problem for higher order p-evolution equations

We consider evolution equations with degree of evolution $p \geq 2$ and order $m \geq 2$; coefficients of the equations are complex valued and depend both on time and space variables; we assume the equations to have real characteristics. We obtain well posedness in H^∞ of the Cauchy problem by giving decay conditions on the coefficients as the space variable $|x| \to \infty$.

Dealing with $p \geq 2$-evolution equations with complex coefficients, the necessity of giving some decay conditions at infinity to get a well posed Cauchy problem in H^∞ arises from [5]. Here we give sufficient conditions for well posedness following the technique of [6, 4].

The results presented in this talk can be found in the recent papers [1, 2, 3].

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Rational Decay Estimates for Fluid-Structure PDE Models

In this talk, we shall demonstrate how delicate frequency domain relations and estimates, associated with coupled systems of partial differential equation models (PDE's), may be exploited so as to establish results of uniform and rational decay. In particular, our focus will be upon decay properties of coupled PDE systems of different characteristics; e.g., hyperbolic versus parabolic characteristics. For such PDE systems of contrasting dynamics, the attainment of explicit decay rates is known to be a difficult problem, inasmuch as there has not been an established methodology to handle hyperbolic-parabolic systems. For uncoupled wave equations or uncoupled heat equations, there are specific Carleman's multiplier methods in the time domain, wherein the exponential weights in each Carleman's multiplier carefully take into account the particular dynamics involved, be it hyperbolic or parabolic. But for coupled PDE systems which involve hyperbolic dynamics interacting with parabolic dynamics, typically across some boundary interface, Carleman's multipliers are readily applicable. Given that such coupled PDE systems occur frequently in nature and in engineering applications; e.g., fluid-structure and structural acoustic interactions, there is a patent need to devise broadly implementable techniques by which one can infer uniform decay for a given PDE system. As one particular example, we shall work to conclude uniform decays for structural acoustic dynamics. In these PDE models, the structural component is subjected to a structural damping ranging from viscous (weak) to strong (Kelvin-Voight). The rational decay rates we derive for this problem explicitly reflect the extent of the damping which is in play. Since the damped elastic component of the coupled dynamics is present on only a portion of the boundary, there will necessarily be assumptions imposed upon the geometry.

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On a Progress in The Theory of Variable Exponent Amalgam Spaces

In this study, we give a historical background of variable exponent amalgam spaces, in particular we focus on harmonic analysis and the boundedness of maximal operators under some conditions in these spaces. Also, we argue and generalize that some inclusions and embeddings properties of classical amalgam spaces.

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Analysis of extended two-operator boundary-domain integral equations for variable-coefficient Dirichelet BVP

Applying a version of the two-operator approach differing from the one considered in [1], the Dirichlet boundary value problem for a second-order scalar elliptic differential equation with variable coefficient and with right-hand side from $\widetilde{H}^{-1}(\Omega)$, is reduced to two different systems of Boundary–Domain Integral Equations, BDIEs. It is proved that both two-operator BDIE systems are equivalent to the boundary value problem, BDIE solvability and invertibility of the boundary-domain integral operators are also proved in the appropriate Sobolev spaces.

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Spaces with generalised smoothness in summability problems for $\Phi-$means of spectral decompositions

In this paper, conditions for localisation of $\Phi-$means of spectral decomposition by system of fundamental functions of Laplace operator are established in terms of belongingness of the being decomposed function to Nikol'skii type spaces with generalised smoothness. This work extends our publications in [1] and generalizes results in [2].

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The Schottky-Klein prime function

In this survey talk I will describe the Schottky-Klein prime function and the Schottky double. Classical Schottky groups and θ_2-series Poincaré is used to solve Riemann-Hilbert problems for n-connected circular domains.

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On one Dubinin's problem

Let $\mathbb{N}$, $\mathbb{R}$ be a set of natural and real numbers, respectively, $\mathbb{C}$ be a complex plane, $\overline{\mathbb{C}} = \mathbb{C}\bigcup\{\infty\}$ be a one point compactification of $\mathbb{C}$, $\mathbb{R}^+ = (0,\infty)$, $\chi(t) = \frac{1}{2}(t + t^{-1})$. Let $r(B, a)$ be a inner radius of domain $B \subset \overline{\mathbb{C}}$ with respect to a point $a \in B$ (see [1, 2]).

Let $n \in \mathbb{N}$. A system of points $A_n := \left\{a_k \in \mathbb{C} : k = \overline{1, n}\right\}$, is called n-***radial***, if $|a_k| \in \mathbb{R}^+$ and $k = \overline{1, n}$, $0 = \arg a_1 < \arg a_2 < \ldots < \arg a_n < 2\pi$.

Denote $\alpha_k := \dfrac{1}{\pi} \arg \dfrac{a_{k+1}}{a_k}$, $\alpha_{n+1} := \alpha_1$, $k = \overline{1, n}$, $\sum\limits_{k=1}^{n} \alpha_k = 2$.

For any n-radial system of points $A_n = \{a_k\}_{k=1}^n$ and $\gamma \in \mathbb{R}^+ \cup \{0\}$ we assume that

$$\mathcal{L}^{(\gamma)}(A_n) := \prod_{k=1}^{n} \left[\chi\left(\left|\frac{a_k}{a_{k+1}}\right|^{\frac{1}{2\alpha_k}}\right)\right]^{1-\frac{1}{2}\gamma\alpha_k^2} \cdot \prod_{k=1}^{n} |a_k|^{1+\frac{1}{4}\gamma(\alpha_k+\alpha_{k-1})}.$$

Theorem. Let $n \in \mathbb{N}$, $n \geqslant 5$, $\gamma \in (0, \gamma_n]$, $\gamma_n = n^{0,38}$. Then for any n-radial system of points $A_n = \{a_k\}_{k=1}^n$ such that $\mathcal{L}^{(\gamma)}(A_n) = 1$, $\mathcal{L}^{(0)}(A_n) \leqslant 1$, and any system of non-overlapping domains B_k, $a_k \in B_k \subset \overline{\mathbb{C}}$, $a_0 = 0 \in B_0$, $(k = \overline{1, n})$, we have the inequality

$$r^{\gamma}(B_0, 0) \prod_{k=1}^{n} r(B_k, a_k) \leqslant r^{\gamma}(D_0, 0) \prod_{k=1}^{n} r(D_k, d_k),$$

where D_k, d_k, $k = \overline{0, n}$, $d_0 = 0$, are circular domains and poles of the quadratic differential $Q(w)dw^2 = -\frac{(n^2-\gamma)w^n+\gamma}{w^2(w^n-1)^2}\, dw^2$. This theorem generalizes the results of papers [3, 4].

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Asymptotic approximations of a thin elastic beam with Robin condition

We start with 3D linear elasticity in a thin beam with anisotropic Robin condition on a part of boundary. The relative thickness of the beam, $\varepsilon = \frac{\text{radius}}{\text{length}}$, is small. This induces a bad conditioning for a direct numerical 3D computation. Therefore we reduce the dimension of the problem with an asymptotic approach following [2]. The Robin parameters are scaled differently in the longitudinal and cross-sectional directions. The 3-D Robin conditions result into 1-D Robin boundary conditions for corresponding ODEs, see [1]. We slightly modify the scaling of the volume forces proposed in [1], extend the asymptotic approximation to the second order and estimate the approximation error. From the mechanical point of view the second order extension is necessary to approximate thicker beams because the second order model includes shear effects, which are present for thicker beams see [3], but are not included in the zeroth order model obtained in [1].

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A fast algorithm to determine the flux around closely spaced non-overlapping disks

This talk is devoted to application of the fast algorithm to determine the flux around closely spaced non-overlapping disks on the conductive plane. This method is based on succesive approximations applied to functional equations. When the distances between the disks are sufficiently small, convergence of the classical method of imagies fails numerically. In this talk, the limitations on geometric parameters of the fast method are described.

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The existence of solution for hemivariational inequality involving $\mathbf{p(x)}$*-Laplacian with the sign-changing weight*

We consider a nonlinear elliptic differential inclusions with p(x)-Laplacian and with Dirichlet boundary condition. They are derived with the help of subdifferential in the sense of Clarke.

In particular, we will consider the following problem

$$(1)\qquad \begin{cases} -\Delta_{p(x)}u(x) - \lambda|u(x)|^{p(x)-2}u(x) \in \partial j(x, u(x)) & \text{a.e. in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with the smooth boundary $\partial\Omega$, $p : \overline{\Omega} \to \mathbb{R}$ is a continuous function satisfying $p(x) > 1$ for all $x \in \Omega$. The function $j(x,t)$ is locally Lipschitz in t-variable and measurable in x-variable. By $\partial j(x,t)$ we denote the subdifferential with respect to the t-variable in the sense of Clarke.

We provide the necessary conditions for the existence of a solution to problem (1) in the situation when λ change the sign. It is a lot of papers with λ negative. There are also some results in the situation when λ is positive, but in some small intervals or with a lot of restrictions on index p such as $p^+ < N$ or $\sqrt{2}p^- > N$. This assumption is necessary to demonstrate the compactness of the embedding of the Sobolev space $W^{1,p(x)}(\Omega)$ into the spaces $C^0(\Omega)$ and $L^\infty(\Omega)$.

Now we significantly expand the class of considered functions because we claim that $\lambda \in \mathbb{R}$. By using the Ekeland variational principle and the properties of variational Sobolev spaces, we establish conditions which ensure the existence of a solution for our problem.

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Existence of a positive solution of discrete equations of (k+1)st order

We use the following notation: for integers s, q, $s \le q$, we define $\mathbb{Z}_s^q := \{s, s+1, \dots, q\}$ where $s = -\infty$ and $q = \infty$ are admitted, too.

The topic of our study is a linear scalar discrete equation

$$\Delta u(n) = f(n, u(n), u(n-1), \dots, u(n-k)),$$

where $f : \mathbb{Z}_a^\infty \times \mathbb{R}^{k+1} \to \mathbb{R}$ and $k \ge 1$ is an integer.

Sufficient conditions are derived for the existence of at least one positive solution

$$u = u(n) > 0, \ n \to \infty$$

of given equation.

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C^* algebras of singular integral operators with shifts

Representations on Hilbert spaces for nonlocal C*algebras of singular integral operators with piecewise slowly oscillating coefficients extended by unitary shifts operators associated with discrete amenable groups of homeomorphisms are constructed. Fredholm symbol calculi are established. The talk is based on joint works with C.A. Fernandes and Y. Karlovich.

[1] M.A. Bastos, C.A. Fernandes, and Yu.I. Karlovich, *Spectral measures in C^*-algebras of singular integral operators with shifts*, J. Funct. Anal. **242** , 86-126 (2007).
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Commutative Banach algebras generated by Toeplitz operators:
structural results and applications

We study new classes of commutative Banach algebras that are generated by Toeplitz operators acting on the standard weighted Bergman space $\mathcal{A}^2_\lambda(\mathbb{B}^n)$ over the complex n-dimensional unit ball $\mathbb{B}^n$ in $\mathbb{C}^n$. These algebras are induced by certain abelian subgroups of the automorphism group of $\mathbb{B}^n$ and only given in terms of their generators. Moreover, they are not invariant under the involution of $\mathcal{L}(\mathcal{A}^2_\lambda(\mathbb{B}^n))$ and cannot be extended to commutative C^*-algebras. The aim of this talk is to describe the Gelfand theory of the above type of algebras that are subordinate to the quasi-elliptic group of automorphisms of $\mathbb{B}^n$. More precisely, we characterize the maximal ideal spaces and provide the Gelfand transform on a dense sub-algebra. These algebras are not semi-simple and in some cases the radical can be calculated explicitely. Finally, we point out that these observations lead to various applications in the spectral theory of Toeplitz operators and can be applied to a further structural analysis of the algebras, e.g. we partly can prove their spectral invariance which typically is not easy to obtain. The results presented in this talk are joint work with Nikolai Vasilevski (CINVESTAV, Mexico).

[1] W. Bauer, N. Vasilevski, *On the structure of a commutative Banach algebra generated by Toeplitz operators with quasi-radial quasi-homogeneous symbols*, Integr. Equ. Oper. Theory 74 (2012), 199-231.

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The parqueting-reflection principle

The parqueting-reflection principle allows to construct certain kernel functions for a class of plane domains which are used to solve boundary value problems for complex partial differential equations. Those kernel functions are the Schwarz kernel, the harmonic Green and Neumann functions providing integral representaion formulas related to the Cauchy-Riemann and the Laplace differential operators. These represention formulas give rise to solutions of the Schwarz, the Dirichlet and the Neumann boundary value problem. Even Robin boundary value problems can be treated for the Poisson equation. By an iteration process higher order equations, like the polyanalytic and the polyharmonic equations, can also be considered. The parqueting-reflection principle can be applied to domains the boundary of which is composed by arcs from lines and circles and which by being consecutively reflected at their boundary parts provide a parqueting of the entire complex plane possibly with the exception of finitely many points. Such domains are e.g. circles, half planes, circle sectors, cones, rings, ring sectors, some convex polygons like rectangles, equilateral triangles, hexagons, half hexagons, lenses, lunes, etc. The talk will mainly present results for a circle sector which leads to a double parqueting of the complex plane and to certain lens and lunes which are composed by the intersection of two orthogonal circles.

[1] Begehr, H. and Vaitekhovich, T., *Harmonic Dirichlet problem for some equilateral triangle*, Complex Var., Ell. Eqs. 57(2012), 185–196.
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[3] Begehr, H. and Vaitekhovich, T., *The parqueting-reflection principle for constructing Green functions*, Preprint, 2013.
[4] Begehr, H. and Vaitekhovich, T., *Schwarz problem in lens and lune*, Preprint, 2013.

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Order of approximation of Besov classes in the metric of anisotropic Lorentz spaces

Let $\mathbf{d} = (d_1, \dots, d_n) \in \mathbb{N}^n$, $\mathbb{T}^{\mathbf{d}} = \{\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_n) : \mathbf{x}_i = (x_1^i, \dots, x_{d_i}^i) \in [0, 2\pi)^{d_i}, i = 1, ..., n\}$. Let $1 < \mathbf{p} = (p_1, \dots, p_n) < \infty$, $1 \leq \mathbf{r} = (r_1, \dots, r_n) \leq \infty$, $\mathbf{0} < \alpha = (\alpha_1, \dots, \alpha_n) < \infty$ and $\mathbf{1} \leq \mathbf{q} = (q_1, \dots, q_n) \leq \infty$. Let $L_{\mathbf{pr}}(\mathbb{T}^{\mathbf{d}})$ be an anisotropic Lorentz space (see [1]) and $B_{\mathbf{pr}}^{\alpha\mathbf{q}}(\mathbb{T}^{\mathbf{d}})$ be a Besov type space (see [2]).

Let $\gamma = (\gamma_1, \dots, \gamma_n)$, $\mathbf{s} = (s_1, \dots, s_n)$, where $\gamma_j > 0$, $s_j \in \mathbb{Z}_+$ for all $j = 1, \dots, n$ and

$$Q^n(\gamma\mathbf{d}, N) = \bigcup_{(\gamma\mathbf{d},\mathbf{s})<N} \rho(\mathbf{s}), \quad T_{Q^n(\gamma\mathbf{d},N)} = \left\{ t(\mathbf{x}) = \sum_{\mathbf{k}\in Q^n(\gamma\mathbf{d},N)} b_{\mathbf{k}} e^{2\pi i(\mathbf{k},\mathbf{x})} \right\}.$$

Let $E_{\gamma\mathbf{d},N}(f)_{L_{\mathbf{pr}}}$ be the best approximation of $f \in L_{\mathbf{pr}}$ by polynomials from $T_{Q^n(\gamma\mathbf{d},N)}$.

Theorem. *Let* $\mathbf{d} = (d_1, \dots, d_n) \in \mathbb{N}^n$, $\mathbf{0} < \alpha = (\alpha_1, \dots, \alpha_n) < \infty$, $\mathbf{1} < \mathbf{p} = (p_1, \dots, p_n) < \mathbf{q} = (q_1, \dots, q_n) < \infty$, $\mathbf{1} \leq \theta = (\theta_1, \dots, \theta_n), \tau = (\tau_1, \dots, \tau_n), \mathbf{r} = (r_1, \dots, r_n) \leq \infty$, $\alpha_{j_0}/d_{j_0} + 1/q_{j_0} - 1/p_{j_0} = \min\{\alpha_j/d_j + 1/q_j - 1/p_j : j = 1, \dots, n\}$ *and* $\alpha_{j_0}/d_{j_0} + 1/q_{j_0} - 1/p_{j_0} > 0$, $\gamma_j = \frac{\alpha_j/d_j+1/q_j-1/p_j}{\alpha_{j_0}/d_{j_0}+1/q_{j_0}-1/p_{j_0}}$, $1 \leq \gamma_j' \leq \gamma_j$, $j = 1, \dots, n$. *Then*

$$E_{\gamma',N}\left(B_{\mathbf{pr}}^{\alpha\tau}\right)_{L_{\mathbf{q}\theta}} \asymp 2^{-(\alpha_{j_0}/d_{j_0}+1/q_{j_0}-1/p_{j_0})N} N^{\sum_{j\in A\setminus\{j_1\}}(1/\theta_j-1/\tau_j)_+},$$

where $E_{\gamma'\mathbf{d},N}\left(B^{\alpha\tau}_{\mathbf{p}\theta}\right)_{L_{\mathbf{q}\theta}} = \sup\limits_{\|f\|_{B^{\alpha\tau}_{\mathbf{p}\theta}}\le 1} E_{\gamma'\mathbf{d},N}(f)_{L_{\mathbf{q}\theta}}$, $A=\{j : \gamma'_j=\gamma_j, j=1,\ldots,n\}$, $j_1=\min\{j : j\in A\}$, $(a)_+=\max(a,0)$.

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[2] Bekmaganbetov, K. and Orazgaliev, E., *Embedding theorems for Nikol'skii-Besov type spaces*, Abstracts of the international conference "Inverse problems: modeling and simulation - VI", 246-247 (2012).

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On asymptotic properties of a general nonlinear delay differential equation

We discuss the following properties: existence of a global positive solution, persistence, uniform permanence, global asymptotic and exponential stability for the following equation

$$\dot{x}(t)=\sum_{k=1}^{m} f_k(t,x(h_1(t)),\ldots,x(h_l(t)))-g(t,x(t)).$$

Some applications to models of Mathematical Biology are also discussed.

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Clifford Wavelets

Wavelets, frames and associated wavelet transforms have been intensively used in both applied and pure mathematics. They provide together with the related multi-scale analysis essential tools to describe, analyze and modify signals, images or in rather abstract concepts functions, function spaces and associated operators. Especially in image processing monogenic signals used to extract certain features such as amplitude, orientation and phase from an image. To do that properly a monogenic wavelet analysis is used.
In this presentation we will present different approaches to construct monogenic wavelets, i.e. wavelets that are monogenic functions itself. One way to that is to use monogenic (Clifford versions) of known wavelets where the underlying function is replaces by the Clifford version. Here we get Clifford-Hermite wavelets and Clifford-Laguerre wavelets. This has been done in the last decade by Clifford group in Gent. Another way which is more based on a discrete setting was done by Held, Storath and Forster. They constructed monogenic wavelets frames (and curvelets) based on the Riesz transform. The key point here is that a Clifford-Hilbert transform is a monogenic function and it can be constructed by the Stein-Weiss system from an scalar valued function.
All these constructions work well in the whole space or half space. but we need other methods to get wavelets on Lie groups or homogeneous spaces (for example) the sphere. For that we introduce the concept of diffusive wavelets that realizes dilations by an diffusive semi-group and replaces translations by an action of a compact group. We will demonstrate that for the sphere. A combination for this construction and the Stein-Weiss system leads to monogenic wavelets on the sphere and also monogenic wavelets frames.
We close our presentation with an remark on monogenic Gabor frames.

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Maximisers for Sobolev–Strichartz and $X^{s,b}$ estimates for the free Schrödinger propagator

We discuss maximisers in the Sobolev–Strichartz estimates

$$\|e^{it\Delta}f\|_{L^q_tL^p_x} \le C\,\|f\|_{\dot{H}^s} \tag{1}$$

and in the bilinear $X^{s,b}$ estimates

$$\|e^{it\Delta}f_1\,\overline{e^{it\Delta}f_2}\|_{X^{s,b}} \le C\,\|f_1\|_{L^2}\|f_2\|_{L^2} \tag{2}$$

and

$$\|e^{it\Delta}f_1 e^{it\Delta}f_2\|_{X^{s,b}} \le C\,\|f_1\|_{L^2}\|f_2\|_{L^2}. \tag{3}$$

In the above, for a space-time function F on $\mathbb{R}\times\mathbb{R}^d$, and $s,b\in\mathbb{R}$, we are writing

$$\|F\|_{X^{s,b}} := \left\| |\xi|^s \left|\left(\tfrac{\tau}{2} - \left|\tfrac{\xi}{2}\right|^2\right)^b \widetilde{F}(\tau,\xi)\right\|_{L^2_{\tau,\xi}},$$

where $\widetilde{F}$ is the space-time Fourier transform of F.

When $(s,b)=(\frac{2-d}{2},0)$, the optimal constant in (2) was found by Ozawa and Tsutsumi in [1], and they showed that it is attained when f_1 and f_2 are the same centred isotropic gaussian. In the case $(s,b)=(0,\frac{2-d}{4})$, we will show that the optimal constant in (3) is also attained for such gaussians. We will also present some new results regarding the existence/characterisation of *gaussian* maximisers associated with the optimal constant in (1), (2) and (3).

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"THE MATHEMATICAL HERITAGE OF Al-Farabi" BY A.KUBESOV IN MODERN CONDITIONS OF EDUCATION

Audanbek Kubesov's monograph "The mathematical heritage of Al-Farabi" [1], well-known and was praised by foreign scientists [2]. In the book based on published and unpublished scientific manuscripts illuminated in mathematics classification of Al-Farabi, geometry, trigonometry, arithmetic, algebra of Al-Farabi and their application in astronomy and in the theory of mathematical music, the doctrine of probabilities and etc. Our goal is to provide the effective usage of the geometrical construct from the mathematics treatises of Al-Farabi [3], namely the geometrical construction regular polygons when we studying the subjects of mathematics and computer science, applying modern methods of teaching and information technology.

Studying of these tasks can cause great achievements in teaching. For example, besides of the task in the form of algorithms for constructing specific polygons are the ability of algorithms to construct higher-order polygons using a smaller orders. These algorithmic approaches in teaching geometric construction of Al-Farabi system of e-learning didactic means. You can effectively implement and learning mathematics-teaching to construct of polygons and e-learning methods of teaching computer science-learning algorithms to construct specific polygons with the further implementation of the automation of constructing any size polygon. Depending on the goals of learning two subjects can offer different methods of using electronic systems and method of teaching geometric constructions named after Al-Farabi.

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Asymptotic solution of one-dimensional diffusion problem for palladium-carbon nanocomposite: comparison with experiments

Applied physics and engineering require the knowledge of the effective properties of composite materials. When heterogeneity spreads over numerous regions, a detailed analysis and numerical approach are very difficult and even impossible. The natural idea is to substitute a real microscopic problem by an averaged one, or a macroscopic problem, such, that a solution of a real problem differs slightly from the averaged one. A useful technique which allows obtaining accurate results is that of homogenization, cf. [1, 2].

In this communication we propose a model of electric current flow through a one-dimensional palladium-carbon nanostructure, and compare the results of numerical computations with the experimental data. We focus on two aspects: 1) calculation of the current flow through the nanowire model for a given conductivity and voltage applied at the ends of the nanowire, 2) determination of the macroscopic parameters in the nanowire model. Since a nanowire microgeometry is highly complex, the homogenization method may be applied to simplify the numerical computations.

A characteristic feature of the homogenization applied to the one-dimensional problem is that an average coefficient is simply the harmonic mean.

In the paper we present the method of calculation of the electrical conductivity in a material composed of many allotropic forms of carbon. This result is of major significance, particularly in the case of nanocomposite materials, in which it is difficult to determine this parameter experimentally.

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Mathematical Modeling in Engineering: possibilities in teaching Differential Integral Calculus

This work refers to a research project whose empirical data were obtained from the use of mathematical modeling (MM) as a teaching method for Differential Integral Calculus (DIC) in a Civil Engineering course. In Brazilian Engineering courses the discipline DIC lasts approximately 240 hours and emphasizes techniques rather than applications. The project aimed at evaluating the students learning of mathematics and verifying the possibilities of establishing modeling as a teaching method for DIC in a regular degree course. We implemented MM as a teaching method with 50 students in the four disciplines comprising the subject DIC (I, II, III & IV), during two years of the Civil Engineering Course. Some themes and model processes were used to teach modeling and the contents, which enabled students to develop a modeling project that started in the first semester and ended in the fourth DIC discipline. Results show that MM as a teaching and research method in Engineering provoked: (1) interest on the part of students as regards applicability; (2) stimulated classroom participation; (3) increased the number of research projects; and (4) increased the general average of grades in written examinations; thus, resulting in a reasonable reduction in the number of dropouts and failures. Despite these results, some facts may be highlighted: (a) students resistance to the teaching proposal, since it is a process demanding research, creativity and reasoning; (b) the non-continuity of the semester system, as it was difficult for some students to realize a long-term research project; and (c) the absence of interaction among other teachers. These results allowed us to understand that modeling is a path for those who want to study, but it is not medicine for those who are not sure about what path to follow.

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Infinitesimal version of Busemann convexity

A sequential approach to the building of pretangent and tangent spaces to a general metric space (X, d) at a point $p \in X$ was proposed in [1] (see also [2]). Such spaces are metric spaces with a metric depending on the initial metric d and a given normalizing sequence of positive real numbers tending to zero. The points of pretangent spaces are some classes of converging to p sequences from X. Among these points there is a natural marked point corresponding to the constant sequence $(p, p, ...)$.

We describe the infinitesimal structure of general metric spaces with Busemann convex pretangent spaces. To this end, the concept of the Busemann convexity at a point is introduced in terms of middle points (for details, see [3], [4]).

Theorem 1. *Let (X, d, p) be a pointed metric space. If X is Busemann convex at p, then every separable tangent space to X at p is Busemann convex and geodesic.*

Theorem 2. *Let (X, d, p) be a pointed metric space. If all pretangent spaces to X at p are Busemann convex and geodesic, then X is Busemann convex at p.*

The characterization of CAT(0) pretangent spaces via infinitesimal Busemann convexity and Ptolemy's inequality is also given.

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Free boundary fluid-elasticity interactions: sensitivity analysis

We perform sensitivity analysis on the fully nonlinear coupling of Navier-Stokes and elasticity. We linearize the system as a whole, and obtain a new linearized model where the coupling on the common interface is quite different than the usual coupling of linear equations. In particular, we notice the presence of curvature terms and boundary acceleration, which can not be neglected for a correct physical interpretation of the problem [1]. We end the talk with a discussion on the well-posedness analysis of the new linearized system.

[1] L. Bociu and J.-P. Zolésio, *Sensitivity analysis for a free boundary fluid-elasticity interaction*, Evolution Equations and Control Theory Volume 2, Number 1, March 2013, 55-79.

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On the degenerate oblique derivative problem for elliptic second-order equation in a domain with boundary conical point.

We study the behavior of strong solutions to the degenerate oblique derivative problem for linear second-order elliptic equation in a neighborhood of the boundary conical point of a bounded domain.

[1] Bodzioch, M. and Borsuk, M. *On the degenerate oblique derivative problem for elliptic second-order equation in a domain with boundary conical point.* "Complex Variables and Elliptic Equations" DOI: 10.1080/17476933.2012.718339

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The overdetermined Cauchy problem in (small) Gevrey classes

We consider the Cauchy problem for overdetermined systems of linear partial differential operators with constant coefficients, in spaces $\mathcal{E}_\omega(\mathbb{R}^k_t \times \mathbb{R}^n_x)$ of ultradifferentiable functions of Beurling type with possible different scales of regularity in the time variables t and in the space variables x.

The solvability of the Cauchy problem is equivalent to the validity of a Phragmén-Lindelöf principle $PL(\omega)$ on the associated affine algebraic varieties V (cf. [1], [2]).

However, this requirement is not always easy to verify in concrete cases, so that we looked for more handy equivalent conditions.

In [3] we obtained some necessary and/or sufficient conditions for the validity of $PL(\omega)$ for affine algebraic varieties V of dimension 1, by means of Puiseux series expansions on the branches at infinity of V. In the case of one time-variable t (and one or more space-variables x) these necessary and sufficient conditions perfectly fit, so that we obtain a complete characterization of algebraic curves V which satisfy $PL(\omega)$: the order of the class of (small) Gevrey (or C^∞) functions where the Cauchy problem is solvable is strictly related to the exponents and the coefficients of the Puiseux series expansions on the branches at infinity of the algebraic curve V.

This looks quite useful, since Puiseux series expansions can be easily computed by several programs, such as MAPLE, for instance.

Then, in [4], we considered some cases of algebraic varieties of higher dimension. In particular, we obtained a complete characterization of hypersurfaces V that satisfy $PL(\omega)$, when V is the set of zeros of a polynomial $P(\tau, \zeta) \in \mathbb{C}[\tau, \zeta_1, \ldots, \zeta_n]$ of degree 2 with principal part P_2 such that $P_2(1, 0, \ldots, 0) \neq 0$ (non-characteristic case).

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[2] Boiti, C. and Nacinovich, M., *The overdetermined Cauchy problem in some classes of ultradifferentiable functions*, Ann. Mat. Pura Appl. **180**, 81-126 (2001).

[3] Boiti, C. and Meise, R. *Characterizing the Phragmén-Lindelöf condition for evolution on algebraic curves*, Math. Nachr. **284**, 1234-1269 (2011).

[4] Boiti, C. and Meise, R. *The Phragmén-Lindelöf condition for evolution for quadratic forms*, Funct. Approx. Comment. Math. **44.1**, 111-131 (2011).

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On multiple dyadic Hardy and Hardy-Littlewood operators

Let $\bar{x} = (x_1, x_2, \ldots, x_n) \in R^n_+$ and vector $\bar{k} = (k_1, k_2, \ldots, k_n)$ with integer coordinates be such that $2^{k_j} \leq x_j < 2^{k_j+1}$, $j = 1, 2, \ldots, n$. For function $f(\bar{x}) \in L_p(R^n_+)$ $(1 < p < \infty)$ we define the multiple dyadic Hardy-Littlewood and Hardy operators respectively by formulas:

$$B_d(f)(\bar{x}) = 2^{-\sum\limits_{j=1}^{n} k_j} \int\limits_0^{2^{k_1}} \int\limits_0^{2^{k_2}} \ldots \int\limits_0^{2^{k_n}} f(t_1, t_2, \ldots, t_n) dt_1 dt_2 \ldots dt_n$$

$$H_d(f)(\bar{x}) = \sum_{m_1=k_1+1}^{+\infty} \ldots \sum_{m_n=k_n+1}^{+\infty} 2^{-m_1-\ldots-m_n} \int\limits_{2^{m_1}}^{2^{m_1+1}} \ldots \int\limits_{2^{m_n}}^{2^{m_n+1}} f(t_1, \ldots, t_n) dt_1 \ldots dt_n$$

For the one-dimensional case the such operators were considered in [1],[2].

Let $k_1 \geq 0, k_2 \geq 0, \ldots, k_n \geq 0$; $m_1, m_2, \ldots, m_n$ be integers and

$$\bar{I} = I_1 \times I_2 \times \ldots \times I_n = I_{k_1}^{m_1} \times I_{k_2}^{m_2} \times \ldots \times I_{k_n}^{m_n} = \left[\frac{k_1}{2^{m_1}}, \frac{k_1+1}{2^{m_1}}\right) \times \left[\frac{k_2}{2^{m_2}}, \frac{k_2+1}{2^{m_2}}\right) \times \ldots \times \left[\frac{k_n}{2^{m_n}}, \frac{k_n+1}{2^{m_n}}\right)$$

is n-dimensional dyadic parallelepiped. We denote the set of dyadic parallelepipeds by D. By definition function $f \in L_{loc}(R_+^n)$ belongs to the space $BMO_d(R_+^n)$, if

$$\|f\|^* = \sup_{I \in D} \frac{1}{\bar{I}} \int_{I_1} \int_{I_2} \ldots \int_{I_n} |f(x_1, x_2, \ldots, x_n) - f_{\bar{I}}| \, dx_1 dx_2 \ldots dx_n < \infty,$$

where $f_{\bar{I}} = \frac{1}{\bar{I}} \int_{I_1} \int_{I_2} \ldots \int_{I_n} f(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n$, and $|\bar{I}|$ denotes the volume of the dyadic parallelepiped. Function $f \in L(R_+^n)$ belongs to the space $H(R_+^n)$, if the dyadic maximal function

$$M(f)(\bar{x}) = \sup_{\bar{x} \in \bar{I}, \bar{I} \in D} \frac{1}{\bar{I}} \int_{I_1} \int_{I_2} \ldots \int_{I_n} f(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n$$

belongs to the space $L(R_+^n)$. The norm in $H(R_+^n)$ is defined by $\|f\|_{H(R_+^n)} = \|M(f)\|_{L_{R_+^n}}$.

Theorem 1. If $f \in BMO_d(R_+^n)$, then the multiple dyadic Hardy-Littlewood operator B_d is bounded in space $BMO_d(R_+^n)$.

Theorem 2. Multiple dyadic Hardy operator H_d is bounded in space $H(R_+^n)$.

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Interior L_p Estimates for Monogenic Functions in Clifford-type algebras

Roughly speaking interior estimates describe the behaviour of the derivatives of a function near the boundary of a bounded domain. They can be found by boundary integral representations and are possible for different norms in the case of elliptic equations in general. For holomorphic functions the necessary interior estimate follows from the Cauchy Integral Formula in the case of the supremum norm.

In this talk we will show an interior estimation through of the L_p-norm, for monogenic functions with values in the Clifford-type algebra depending on parameters $\mathbf{A}_n(2, \alpha_i, \gamma_{ij})$. This estimation is obtained from a Cauchy-type integral representation, an estimate for the product of functions in $\mathbf{A}_n(2, \alpha_i, \gamma_{ij})$ and an inequality type Hölder in $\mathbf{A}_n(2, \alpha_i, \gamma_{ij})$.

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Biomechanical effects of rapid maxillary expansion in cleft palate patients during Hyrax treatment

One of the anomalies of the upper jaw is a congenital defect (cleft) lip or palate. Various orthodontic appliances could be used for their treatment for example the orthodontic appliances Hyrax [1]. Clinical observations show that the intensity and behavior of teeth and jaw displacements are depended on the design features of devices for maxillary expansion [2]. In this paper, finite element analysis of the movements maxillary bones of the craniofacial skeleton with the unilateral cleft palate for different designs orthodontic appliance is carry out.

Stereolithography (STL) model of the cranium is obtained with the use of software for medical imaging MIMICS. The finite element model is obtained after processing STL-model in 3-matic MIMICS module. Finite element model of orthodontic device, premolars and the first molars are constructed in the ANSYS Workbench. Hyrax model and teeth is added to the finite element model of the skull after importing to Finite Element Modeler.

The boundary conditions of skull correspond to fixed support of nodes in the foramen magnum environment [3].

After finite element calculation the distribution of equivalent stresses and total displacements of the cranium are obtained for seven different designs of orthodontic appliance. Hyrax constructions are differ by have different positions of plates comparatively the palate. In the first case, bars and plates of orthodontic appliance locate in the same plane, in other cases the plate orthodontic appliance locate at 0.5, 1, 2, 4, 6 and 8 mm higher (closer to the palate) to the horizontal position.

The direction and magnitude of total displacement vary significantly when the orthodontic appliance plates are closer to the palate. In the case of horizontal placement of bars and plates total displacement of the upper jaw points differ approximately in half compare with the case when the Hyrax plate raise to 8 mm above with respect to the horizontal position. This difference of maximum total displacement can be explained by significant deformation of the nasal bone and the frontal process of the maxilla for activated orthodontic appliance with the plates close to the palate. Distribution pattern of movement indicates that the best design of the device is the location of the plates at a distance of 0.5 mm from the horizontal plane.

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Atiyah Classes of Lie Algebroids

In [2] Kapranov proved that the Atiyah class of the tangent bundle of a complex manifold X makes $T_X[-1]$ into a Lie algebra object in the derived category $D^+(X)$. After introducing the notion of a $(A,\sharp)$-connection on a holomorphic vector bundle E over X, where $\sharp : A \to T_X$ is a holomorphic Lie algebroid over X, we define the $(A,\sharp)$-Atiyah class of E as the obstruction to the existence of a $(A,\sharp)$-connection on E. We will show that usual constructions like jet bundles, bundles of differential operators, etc., admit natural generalizations in the framework of $(A,\sharp)$-connections. Finally we will prove an analogue of Kapranov's theorem, obtained by replacing the tangent bundle T_X with any holomorphic Lie algebroid over X. Namely, we will prove that for any holomorphic Lie algebroid $\sharp : A \to T_X$, the $(A,\sharp)$-Atiyah class of A makes $A[-1]$ into a Lie algebra object in the derived category.

Similar results have been obtained independently by Z. Chen, M. Stiénon and P. Xu in [1], by using different techniques.

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Generalized shift operators generated by convolutions of integral transforms and their properties

In this work we discuss the generalized shift operators generated by nonclassical convolution constructions of integral transforms. In particular, we consider the shift operators defined by the convolutions for Hankel integral transform with the function $j_\nu(xt) = (2xt)^\nu \Gamma(\nu+1) J_\nu(xt)$ in the kernel. Here $J_\nu(xt)$ is the Bessel function of the first kind of order ν, $\mathrm{Re}\,\nu > -1/2$. We introduce the explicit view of these operators and obtain their properties.

The generalized shift operator generated by classical convolution for Hankel transform was introduced and studied by B.M. Levitan in 1951 and D.T. Haimo in 1965. More recently, it has been studied by J.J. Betancor and co-authors in spaces of generalized functions.

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Global existence of small data solutions for semi-linear damped wave equations

The goal of our talk is to consider the global existence (in time) of small data solutions to the following semi-linear Cauchy problem

$$(1) \qquad u_{tt} - a^2(t)\Delta u + b(t)u_t = f(u, a(t)\nabla u, u_t),\ u(0,x) = u_1(x),\ u_t(0,x) = u_2(x)$$

with an increasing time-dependent speed of propagation term $a^2(t)$ and a time-dependent positive coefficient $b(t)$ in the damping term $b(t)u_t$. We distinguish between two semi-linear models with respect to the following classification of $b(t)$: *non-effective dissipation* and *effective dissipation*. The first model we are interested in is the model with non-effective dissipation and on the right-hand side a source in the form $f(a(t)\nabla u, u_t) = u_t^2 - a^2(t)|\nabla_x u|^2$ (in the case $b(t) \equiv 0$ the results can be found in [2]), whereas in the other model with effective dissipation we consider the right-hand side source in the form $|f(u)| \approx |u|^p$. Here we follow the technique of [3].
The results presented are part of the thesis of Bui Tang Bao Ngoc[1].

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Spectral perturbation for vibrating plate models

We consider two different models for the vibration of a clamped plate: the Kirchhoff-Love model, which leads to the well known biharmonc operator, and the Reissner-Mindlin model, which instead gives a system of differential equations. We point out similarities and differences, showing the connections between these two problems. After recalling the known results in shape optimization for the biharmomic operator, we state some analyticity results for the dependence of the eigenvalues upon domain perturbations and Hadamard-type formulas for shape derivatives. Using these formulas, we will that balls are critical for the symmetric functions of the eigenvalues under volume constraint.

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Interpolation theorems for general local Morrey-type spaces

We consider the real interpolation method and prove that for general local Morrey-type spaces, in the case when they have the same integrability parameter, the interpolation spaces are again general local Morrey-type spaces with appropriately chosen parameters.

Let (Ω,μ) be a space with a positive σ-finite Borel measure μ. By $G=\{G_t\}_{t>0}$ we denote a parametric family of μ-measurable subsets of Ω, for which

$$G_t \neq \Omega \text{ for some } t>0, \quad G_{t_1}\subset G_{t_2} \text{ if } 0<t_1<t_2<\infty \text{ and } \bigcup_{t>0} G_t = \Omega.$$

Definition. Let $0<p,q\le\infty$ and $0<\lambda<\infty$ if $q<\infty$ and $0\le\lambda<\infty$ if $q=\infty$. We define the space $LM^{\lambda}_{p,q}(G,\mu)$ as the space of all functions f μ-measurable on Ω such that

$$\|f\|_{LM^{\lambda}_{p,q}(G,\mu)} = \left(\int_0^\infty \left(t^{-\lambda}\|f\|_{L_p(G_t,\mu)}\right)^q \frac{dt}{t}\right)^{1/q} < \infty.$$

Theorem. *Let* $0<p,q_0,q_1,q\le\infty$, $0<\lambda_0,\lambda_1<\infty$, $\lambda_0\neq\lambda_1$, $0<\theta<1$ *and* $\lambda=(1-\theta)\lambda_0+\theta\lambda_1$. *Then*

$$(LM^{\lambda_0}_{p,q_0}(G,\mu), LM^{\lambda_1}_{p,q_1}(G,\mu))_{\theta,q} = LM^{\lambda}_{p,q}(G,\mu).$$

For details and corollaries see [1]. This result is a particular case of the interpolation theorem for much more general spaces. The classical interpolation theorems due to Stein-Weiss, Peetre, Calderón, Gilbert, Lizorkin, Freitag can also be derived from that theorem.

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The Chern-Simons-Dirac equations in the Coulomb Gauge

The Chern-Simons-Dirac equations are a semi-linear hyperbolic system with a gauge invariance. Somewhat surprisingly, they exhibit an elliptic component independently of the choice of gauge. This is unlike the related Maxwell-Dirac, or Yang-Mills systems which only have an elliptic component in the Coulomb gauge. We show how this elliptic structure can be used together with the Coulomb gauge to obtain local well-posedness at low regularities with only the use of the refined Strichartz estimates of Klainerman-Tataru. In particular we have no need to exploit the null structure of the Chern-Simons-Dirac equations, nor do we use the more complicated $X^{s,b}$ type spaces. This is joint work with Nikolaos Bournaveas and Shuji Machihara.

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Preliminary results on the accuracy of a simple approximate solution for Ph/Ph/c/K queues

A large number of systems with multiple homogenous servers and limited queueing room can be modeled as instances of the Ph/Ph/c/K queue. Phase-type distributions are used to represent general distributions of the times between arrivals and of service times thus leading to more realistic models. The exact analytical solution of such queues is not known so that (outside simulation) the state equations have to be solved numerically. As the number of servers and the number of phases in the service and inter-arrival time distributions increase, the number of equations involved and the memory space needed for the solution increase very rapidly making the solution of many models impractical.

In an attempt to reduce the inherent complexity of the Ph/Ph/c queue, we propose to replace the solution of such a queue by an iteration between the solution of an M/Ph/c queue with state-dependent arrivals and a Ph/M/c queue with state-dependent service. To the extent that the number of iterations needed remains relatively small, this approach has the obvious potential to reduce the complexity of the overall solution process and allow tackling problems with larger numbers of servers and/or phases than currently feasible. This paper presents preliminary results investigating the accuracy of the proposed approach. These results indicate that the relative errors compared to an exact numerical solution appear generally modest.

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Sobolev estimates and spectra for generalized SG-hypoelliptic pseudo-differential operators on $L^p(\mathbb{R}^n)$

The first goal of this talk is to present an analogue of Agmon-Douglis-Nirenberg estimates in the frame of the L^p-Sobolev spaces $H^{s_1,s_2,p}$, $1 < p < \infty$, $-\infty < s_1, s_2 < +\infty$, for a class of generalized SG-hypoelliptic pseudo-differential operators introduced and studied by Camperi in [1]. We also state and prove some results concerning spectra or essential spectra for such kind of SG-hypoelliptic pseudo-differential operators perturbed or not by suitable singular potentials on $L^p(\mathbb{R}^n)$, $1 < p < \infty$. A self-adjointness result is proved for some perturbations of a SG-hypoelliptic pseudo-differential operators on $L^2(\mathbb{R}^n)$, which symbol is independent of $x \in \mathbb{R}^n$. Finally, a perturbation result concerning strongly continuous semigroups of contractions generated by SG-hypoelliptic pseudo-differential operators on $L^p(\mathbb{R}^n)$, $1 < p < \infty$ is given.

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A Boundary Value Problem for Bi-analytic Functions in a Multiply Connected Domain

We consider a boundary value problem for bi-analytic function in $D = \hat{\mathbb{C}} \setminus \bigcup_{k=1}^{n} D_k$ where $D_j \cap D_l = \emptyset$ if $l \neq j$. We employ the equivalent $\mathbb{R}$-linear problems to find the solutions. Particularly, in the case of $n = 2$ we have obtained an explicit solution.

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Towards a discrete function theory: tackling Hardy spaces

In recent years one can observe an increasing interest in obtaining discrete counterparts for various continuous structures. While such ideas are very much developed in the complex case (see e.g. recent results of S. Smirnov in connection with complex discrete function theory) the higher-dimensional case is yet underdeveloped. This is mainly due to the fact that discrete Clifford analysis started effectively only in the eighties and nineties with the construction of discrete

Dirac operators either for numerical methods of partial differential equations or for quantized problems in physics (c.f. [1, 2, 3, 4, 5]). The development of genuinely function theoretical methods only started quite recently (see [6, 7]). In this talk we study the boundary behavior of discrete monogenic functions, i.e. null-solutions of a discrete Dirac operator, in the upper and lower half space. Calculating the Fourier symbol of the boundary operator we construct the corresponding discrete Hilbert transforms, the projection operators arising from them, and discuss the notion of discrete Hardy spaces. Hereby, we focus on the 3D-case with the generalization to the n-dimensional case being straightforward.

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Critical points of random polynomials

In this presentation, I will introduce a quite recent problem lying at an intersection of complex polynomials and probability theory, which can be stated concisely as:

> *For random complex polynomials with independent and identically distributed zeros following any common probability distribution μ on $\mathbb{C}$, it follows readily from Kolmogorov's strong law of large numbers that the empirical measures of these zeros converge weakly to μ almost surely. Does the same weak convergence happen for those of the derived critical points? How probable (e.g. almost surely, in probability)?*

This straightforward question was initiated by Pemantle and Rivin [2] in 2011 (with late O. Schramm's contribution to its preliminary ideas in 2001) as a very first probabilistic framework for studying critical points of large degree polynomials.

In contrast to the long tradition of prescribing probability distributions to coefficients, the randomness of the polynomials in the present problem comes from their zeros, thus opening an alternative path in the theory of random polynomials. On the theory of complex polynomials side, via [2], we can suggest a probabilistic viewpoint to respond to the issue of the locations of the critical points relative to the zeros, which is one type of classical questions in geometry of polynomials (e.g. the well known Gauss–Lucas theorem, and the still open Sendov's conjecture).

There are two follow-ups to [2], namely Subramanian [3] and Kabluchko [1]. I will briefly explain our completion and generalization of [3].

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Development of some analytical methods for calculating of cusped prismatic shells

Natalia Chinchaladze

Cusped plates and beams, on the one hand, are very important details from the practical point of view, such plates and beams are often encountered in spatial structures with partly fixed edges, e.g., stadium ceilings, aircraft wings, submarine wings etc., in machine-tool design, as in cutting-machines, planning-machines, in astronautics, turbines, and in many other areas of engineering (e.g., dams); on the other hand, their theoretical analysis and calculation are mathematically connected with the study of very difficult problems for degenerate partial differential equations which are not covered by the general theory for degenerate partial differential equations (see, e.g., [1], [2]). Some satisfactory results are achieved in this direction in the case of Lipschitz domains but in the case of non-Lipschitz domains there are a lot of open problems. To investigate such open problems is a main part of the the present talk. To this end there are used function-analytic, approximate and special methods (suitable to problems peculiarities). As results, boundary value problems in the zero approximation for I.Vekuas hierarchical models of cusped plates and beams in case of 3-D non-Lipschitz domains will be investigated.

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Parametric Continuity of Choquet and Sugeno Integrals

For a fixed function f and a fixed set A, the continuity with respect to the real parameter λ of the Choquet or Sugeno integral $\int_A f dm(\lambda)$ is proved. Here $m(\lambda)$ are all possible $\lambda-$measures generated by a fixed classical measure.

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Critical point theory and the asymmetric beam system

We show the existence of at least two solutions for a class of systems of the critical growth nonlinear suspension bridge equations with Dirichlet boundary condition and periodic condition. We first show that the system has a positive solution under suitable conditions, and next show that the system has another solution under the same conditions by the linking arguments.
Key Words and Phrases: System of the critical growth suspension bridge equations, linking arguments, eigenvalue of a matrix, boundary value problem.

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On Inequalities with Alternating Signs

Most known inequalities are essentially concerned with non-negative sequences. Inequalities with alternating signs were first obtained by G. Szegö (1950), H. F. Weinberger (1952), R. Bellman (1953). Further development was due to H. D. Brunk (1956), I. Olkin (1959), R. Bellman (1959) (see [1] and [2, §§49-52]), who considered inequalities of Jensen type.

Our aim is to obtain inequalities of Hölder, Minkowski and Hardy type for sums with alternating sign. Let us formulate some results.

We assume that the sequences $\{a_k\}$ and $\{b_k\}$ are non-negative, monotone decreasing, and

$$0 < a \le a_k \le A < \infty, \qquad 0 < b \le b_k \le B < \infty \quad \forall k = \overline{1,n}.$$

Theorem 1. *Let $p > 1$ and $1/p + 1/q = 1$. Then the following inequality of Hölder type holds*

$$\left(\sum_{k=1}^{n}(-1)^{k+1}a_k^q\right)^{1/q}\left(\sum_{k=1}^{n}(-1)^{k+1}b_k^p\right)^{1/p} \le \frac{Ab+aB}{ab}\sum_{k=1}^{n}(-1)^{k+1}a_kb_k.$$

In case of $p = q = 2$ we can obtain an inequality with a better constant if the sequences $\{a_k\}$ and $\{b_k\}$ satisfy additional conditions.

Theorem 2. *Let the sequence $\{a_k/b_k\}$ be monotone. Then the following inequality of Cauchy-Bunyakovsky-Schwarz type holds*

$$\sqrt{\sum_{k=1}^{n}(-1)^{k+1}a_k^2\sum_{k=1}^{n}(-1)^{k+1}b_k^2} \le \frac{1}{2}\max\left\{\frac{a^2+A^2}{aA};\frac{b^2+B^2}{bB}\right\}\sum_{k=1}^{n}(-1)^{k+1}a_kb_k.$$

Equality holds if $a_k = A = a$ and $b_k = B = b$.

This research is partially supported by MTM 2011-27637 grant.

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Extinction and positivity for the evolution p-Laplacian equations with absorption on networks

In recent years, the discrete p-Laplacian $\Delta_{p,\omega}$ on networks (or weighted graphs) has attracted many researcher's attention not only because it is a useful tool for modeling nonlinear phenomena on discrete media (see [1, 2, 3, 4, 5, 6] and references therein) but also because it can be considered as the discrete analogue of the p-Laplacian on Riemannian manifolds (for a physical meaning of the p-Laplacian on continuum, see [7]).
In this lecture, we discuss the extinction and positivity of solutions of evolution p-Laplacian equations with absorption $u_t - \Delta_{p,\omega} u + |u|^{q-1}u = 0$ on finite networks with $p > 1$ and $q > 0$. We obtain the necessary and sufficient conditions for extinction and positivity.
More precisely, it is proved that the solution is positive if $p \geq 2$ and $q \geq 1$, whereas the solution becomes extinct in finite time if $1 < p < 2$ or $0 < q < 1$. In addition, it is proved that the solution decays either exponentially, or polynomially but not exponentially, according to the conditions p, q when the solution is positive, and an estimate for the extinction time is derived if the solution become extinct in finite time.

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Recent results on the F-functional calculus

In the recent years the theory of slice hyperholomorphic functions has become an important tool to study functional calculi for n-tuples of operators. In particular, using the Cauchy formula for slice hyperholomorphic functions it is possible to give the Fueter mapping theorem an integral representation. Thanks to this integral representation it has been defined a monogenic functional calculus for n-tuples of bounded commuting operators, the so called F-functional calculus, which is based on the notion of F-spectrum and of F-resolvent operator. Using this resolvent operator we have introduced a new notion of convergence for an n-tuples of operators. Moreover, we show that it is possible to define this calculus also for n-tuples of unbounded operators and we obtain an integral representation formula. As we will see, it is not an easy task to provide the correct definition of F-functional calculus in the unbounded case.

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Application of symbolic-numerical calculation to the determination of effective conductivity of random two-dimensional composites with non-overlapping circular inclusions.

We study the effective conductivity of equal unidirectional infinite circular cylinders randomly distributed in a uniform host (disks on the plane). The problem is reduced to a boundary value problem for the two–dimensional Laplace equation. A symbolic–numerical algorithm was proposed in the previous papers to solve the boundary value problem with arbitrary deterministic locations of disks. Application of the Monte Carlo method for the uniform non–overlapping distribution of disks yields the effective conductivity of random composites. The expected value of the effective conductivity is written exactly in the form of a power series in the concentration and Bergman's contrast parameter. This formula is valid for all concentrations.

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Canonical divisors on surfaces of general type

One of the applications of Reider's theorem is the classical result of Bombieri to the effect that the linear system $|mK_X|$ is base point free for $m \geq 4$. This implies, in particular, that the base point freeness of pluricanonical maps is determined numerically. We investigate here to what extend the assumptions in the Reider's theorem are optimal and whether the existence of base loci of pluricanonical systems is numerically determined also for small values of m. In order to do this, we consider two linear systems: $|3K_X|$ and $|2K_X|$.

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An application of $L^p - L^q$ decay estimates to the semilinear wave equation with structural parabolic-like damping

In this talk we discuss the global existence of the small data solution to

$$u_{tt} - \triangle u + 2a(-\triangle)^\sigma u_t = |u|^p\,, \qquad u(0,x) = u_0(x)\,, \quad u_t(0,x) = u_1(x)\,,$$

where $\sigma \in (0, 1/2]$ and $p > 1$. Assuming small data in some Sobolev spaces, we obtain global existence for $p > 1 + 2/(n - 2\sigma)$, in space dimension $n \leq \overline{n}$, where $\overline{n} = \overline{n}(\sigma) \nearrow \infty$, as $\sigma \to 1/2$. In particular, $\overline{n}(1/2) = \infty$. Thanks to the use of $(L^p \cap L^q) - L^q$ estimates [1, 2], not on the conjugate line, which generalize the $L^q - L^q$ estimates derived in [4], we may extend some previous result obtained in [3].

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The Cauchy Problem for the Stokes Equations

In the domain $\Omega = \{(x,y) \in R^2 : x \in (-a,a), y \in (\psi_1(x), \psi_2(x))\}$ consider the Cauchy problem for the Stokes system

(1) $$\Delta u - \nabla p = 0,$$

(2) $$div\, u = 0,$$

(3) $$u = \varphi,\ (x,y) \in \Gamma_0,$$

(4) $$pn - \frac{\partial u}{\partial n} = f,\ (x,y) \in \Gamma_0,$$

where $u = (u_1, u_2))$, $\partial\Omega = \Gamma_0 \cup \Gamma_1$ is the boundary of the domain Ω, $\Gamma_1 = \{(a,y) : y \in [\psi_1(a), \psi_2(a)]\}$, $n = (n_1, n_2)$ is the outward unit normal to $\partial\Omega$, $\varphi \in H_{00}^{1/2}(\Gamma_0)$, $f \in \left(H_{00}^{1/2}(\Gamma_0)\right)^\star$. This spaces are consided in [1]. The problem (1)-(4) is ill-posed. It can be formulated as the inverse problem for some direct well-posed problem. Analogous approach for other problems may be found in [2].

Consider the problem (1)-(3) with the following condition

(5) $$pn - \frac{\partial u}{\partial n} = q, \quad (x,y) \in \Gamma_1.$$

The direct problem: given the function $q = (q_1, q_2)$ on Γ_1, determine the solution (u,p) of the problem (1)-(3), (5). This problem is well-posed.

Then the initial problem (1)-(4) is reduced to the following inverse problem: it is required to determine q on Γ_1 from the additional information (4) about the solution of the direct problem (1)-(3), (5).

Consider the operator

$A : \quad q := (pn - \frac{\partial u}{\partial n})|_{\Gamma_1} \to f := (pn - \frac{\partial u}{\partial n})|_{\Gamma_0}$, where (u,p) is the solution of the direct problem (1)-(3), (5). Then the inverse problem (1)-(3), (5), (4) can be written as an operator equation

(6) $$Aq = f,$$

where $A : \left(H_{00}^{1/2}(\Gamma_1)\right)^\star \to \left(H_{00}^{1/2}(\Gamma_0)\right)^\star$, f is the given function, q is unknown function. The inverse problem (6) numerical is solved on the bases combination of the finite method element end the optimization method.

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Multi-quasielliptic and Gevrey regularity of hypoelliptic differential operators

A large class of hypoelliptic linear differential operators with constant complex coefficients is considered, the so called multi-quasielliptic differential operators, see [5] for an exhaustive study.

We first show the theorem of L. Zanghirati [11] and C. Bouzar and R. Chaïli [1] only in the case of the linear differential operators with constant complex coefficients.

Due to a result of Hörmander [8] every hypoelliptic linear partial differential operators with constant coefficients is Gevrey hypoelliptic in some anisotropic Gevrey spaces. We first explicit this anisotropic Gevrey regularity result for multi-quasielliptic differential operators.

Zanghirati [11] has also proved that there exists a Gevrey regularity result for multi-quasielliptic differential operators in the context of the so called multi-anisotropic Gevrey spaces.

We prove that the multi-anisotropic Gevrey regularity result is more precise than the anisotropic Gevrey regularity for multi-quasielliptic differential operators. An illustrative example is given.

The aim of this paper is to prove the multi-anisotropic Gevrey regularity of hypoelliptic linear differential operators with complex constant coefficients and consequently we precise the result of L. Hörmander and extend the result of L. Zanghirati.

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Dirichlet problem in a planar domain with a small hole

We consider a Dirichlet problem in a planar domain with a hole of diameter proportional to a real parameter ϵ and we denote by u_ϵ the corresponding solution. The behavior of u_ϵ for ϵ small and positive can be described in terms of real analytic functions of two variables evaluated at $(\epsilon, 1/\log\epsilon)$. We show what happens when the parameter ϵ is negative. We also show that under suitable assumptions one can get rid of the logarithmic behavior displayed by u_ϵ for ϵ small and describe u_ϵ by real analytic functions of ϵ. The results presented continue the work of [1], where the Dirichlet problem in a perforated domain of $\mathbb{R}^n$, with $n \geq 3$, has been investigated. Here instead we focus on the two-dimensional case.

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On the Helmholtz equation on exterior domains

In a series of joint works with F.Cacciafesta, R.Luca' and B.Cassano (Postdoc and Ph.D. students) we consider an Helmholtz equation with fully variable coefficients and lower order terms on the exterior of a star-shaped domain in $\mathbb{R}^n$ $(n \geq 3)$ with Dirichlet boundary conditions. We prove a sharp weighted L^2 estimate of solutions under weak conditions on the coefficients; in particular, the metric of the principal part of the opreator is not required to be flat at infinity in dimension $n \geq 4$ (while it can be a long range perturbation of identity in dimension $n = 3$). We also remark that no nontrapping condition is imposed, but only explicit assumptions in physical space variables. Applications include smoothing estimates for Schrödinger and wave equations with fully variable coefficients, and scattering for nonlinear Schrödinger equations via the interaction Morawetz technique.

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GEVREY FUNCTIONS AND ULTRADISTRIBUTIONS ON COMPACT LIE GROUPS AND HOMOGENEOUS SPACES

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Split functions, Fourier transform and multipliers

We study the effect of a splitting operator S_t on the L^p norm of the Fourier transform of a function f and on the operator norm of a Fourier multiplier m. Most of our results assume p is an even integer, and are often stronger when f or m has compact support. Some of the result are in a joint paper with S. Hufdson

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Schatten classes and $r-$nuclearity on Compact Lie groups

In this work we study some ideals of operators on Compact Lie groups. We start by a characterisation of invariant operators in Schatten classes. As an application we consider negative powers of some differential operators on compact Lie groups. In particular the case of powers of sublaplacians is explained as well as powers of Schrodinger operators. Secondly we present sufficent conditions for $r-$nuclearity on L^p spaces and some applications to Lidskii formula and distribution of eigenvalues. It is also put in evidence how the concept of $r-$nuclearity is closely related to the discrete decomposition of kernels with respect to the discrete unitary dual. The criterias are given in terms of symbols. The main idea consist in exploiting the notion of global matrix-symbol.

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On a problem of estimation of product of generalized inner radii in n-dimensional complex space.

A purpose of this work is to study a problem of finding the product of powers of generalized conformal radii poly-cylindrical non-overlapping domains. This problem relates to a number of problems with so-called "free" poles (see, for example, [1, 2]). Dimensional analogues of some well-known results on non-overlapping domains on the plane were obtained in [3]. For this in the [3] the concept of inner radius was generalized, namely, a concept of harmonic radius of the space domain $B \subset \mathbb{R}^n$ with respect to some internal point was introduced. In this work an approach is proposed that allows to transfer some results known in the case of the complex plane on the space $\overline{\mathbb{C}}^n$. Another approach was proposed in [3].

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Compulsory behavior of solutions of dynamic equations on time scales

Let b_i, $c_i\colon \mathbb{T} \to \mathbb{R}$, $i = 1, \dots, n$ be delta differentiable functions, $b_i(t) < c_i(t)$, $t \in \mathbb{T} \subset [t_0, \infty)$ where $\mathbb{T}$ is a time scale. Let $\Omega \subset \mathbb{T} \times \mathbb{R}^n$, $\Omega := \{(t, u)\colon t \in \mathbb{T}, u \in \omega(t)\}$ and (for $t \in \mathbb{T}$)

$$\omega(t) := \{u \in \mathbb{R}^n\colon b_i(t) < u_i < c_i(t),\ i = 1, \dots, n\}.$$

We consider a dynamic system

$$u^\Delta = f(t, u)$$

where $f\colon \mathbb{T} \times \mathbb{R}^n \to \mathbb{R}^n$ and f is continuous. Moreover, for every fixed non-isolated point $t \in \mathbb{T}$, let $S(t) \subset \mathbb{T} \times \mathbb{R}^n$ be a closed set, $[t-a, t+a] \cap \overline{\Omega} \subset S(t)$ for an $a > 0$, $\inf \mathbb{T} \le t-a$, $\sup \mathbb{T} \ge t+a$, such that f is rd-continuous, bounded and Lipschitz continuous on $S(t)$; if $t = t_0$ is non-isolated, we define the set $S(t_0)$ in a similar way. Let $t \in \mathbb{T}$, $t > t_0$ and

$$u(t) = u^* \in \overline{\omega}(t).$$

Sufficient conditions are derived for the existence of at least one solution such that $(t, u(t)) \in \Omega$ for each $t \in \mathbb{T}$.

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On a p-adic invariant cycles theorem

Let $\mathcal{X}$ be a smooth complex manifold, $f : \mathcal{X} \to D$ a projective morphism and D the unit disk in the complex plane. We suppose that every fiber of $\mathcal{X}$ is smooth except $f^{-1}(0) := \mathcal{X}_0$, which is a divisor with normal crossing. Steenbrink [4] defined a limit cohomology $H_{\lim}$ endowed an operator which is (the logarithm of) the monodromy operator. The invariant cycles theorem says that every element in $H_{\lim}$ which is invariant under the action of the monodromy operator comes from an element of the cohomology of $\mathcal{X}_0$. In a joint work with B. Chiarellotto, R. Coleman and A. Iovita [2] we study a p-adic analogue of this theorem. Let X be a proper semistable curve over a DVR, *i.e* the special fiber is a normal crossing divisor and the generic fiber is smooth. The limit cohomology on which the monodromy operator acts is given in this case by the Hyodo-Kato cohomology [3]. We give a new proof of a theorem of Chiarellotto ([1]) which says that the kernel of the monodromy operator, acting on the first Hyodo-Kato cohomology group, coincides with the first rigid cohomology group associated to the special fiber. We also analyze the case of cohomologies with nontrivial coefficients.

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Multi-metamonogenic function in different dimension

A metamonogenic of first order function or simply metamonogenic function is a function that satisfies the differential equation $(D-\lambda)u=0$, where D is the Cauchy-Riemann and $\lambda\in\mathbb{R}$. Using this definition we can say that a multi-metamonogenic function u is separately metamonogenic in several variables x^j , $j=1,\dots n$ with $n\leq 2$, if $x^j=(x_1^{(j)},\dots x_{m_j}^{(j)})$ runs in the Euclidean space $\mathbb{R}^{m_j}$, that is $(D_j-\lambda)u=0$, for each $j=1,\dots,n$,. Using the theory of algebras of Clifford type depending on parameters (see [4, 5]), the present proposal discusses the properties of u in case the dimensions m_j are different from each other following the ideas exhibited in [2].

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Robin function and conformal welding a new proof of existence

Green's function is a powerful tool for solving boundary value problems of potential theory. Depending on the boundary problem there are three kinds of Green's function. Green's function of the mixed boundary-value problem for harmonic function is sometimes called Robin function.

Let $\Omega\ni\infty$ be a domain of connectivity n in the extended complex plane $\hat{C}$ with the boundary $\partial\Omega=C=\cup_{i=1}^n C_i$, where C_i are simply closed Jordan curves.

On C_i there may be given $m_{ij}, i=1,...,n, j=1,...,m_{ij}$ closed arcs A_{ij}, which can also be equal to C_i, it is also possible that no arc A_{ij} is given on C_i, $A=\cup_{i=1}^n\cup_{j=1}^{m_{ij}}A_{ij}$.

The Robin function $R_{\Omega,A}(z,\zeta)$ of the domain Ω with respect to the boundary set A is defined by the following properties:
1.$R_{\Omega,A}(z,\zeta)$ is harmonic in Ω and continuous in $\overline{\Omega}$, except at $z=\zeta$, where $R_{\Omega,A}(z,\zeta)+\log|z-\zeta|$ is harmonic, for $\zeta=\infty$ this is modified to require that $R_{\Omega,A}(z,\zeta)-\log|z|$ be harmonic in Ω,
2. $R_{\Omega,A}(z,\zeta)=0$ for all $z\in A$,
3. $\frac{\partial R_{\Omega,A}(z,\zeta)}{\partial n}(z,\zeta)=0$ for all $z\in B$, where n denotes the inner normal.
P. L. Duren and M. .M. Schiffer [1, 2, 3] found among other properties of the Robin function a way to prove the existence of the Robin function for a simply connected domain Ω. The aim of this talk is a new proof of existence for the case that Ω is n-fold connected basing on conformal welding.

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On a type of generalized moduli of smoothness and their application

The talk presents a method for constructing an unweighted fixed-step modulus of smoothness, which is equivalent to a weighted K-functional. The approach is applicable to a very broad class of K-functionals, including such as

$$K(f,t) = \inf_{g\in AC_{loc}^{r-1}} \{\|w_1(f-g)\|_{L_p[a,b]} + t^r \|w_2 g^{(r)}\|_{L_p[a,b]}\}$$

for power-type weights w_1 and w_2 with singularities only at the ends of the interval with arbitrary exponents, as well as

$$K(f,t) = \inf_{g\in AC^{r-1}} \{\|f-g\|_{L_p[a,b]} + t^r \|Lg\|_{L_p[a,b]}\},$$

where L is a linear differential operator of order r, with constant leading coefficient and generally non-constant but smooth lower coefficients.

The method is based on relating two K-functionals by means of a continuous linear transform of the function.

We consider applications in defining moduli which characterize the error of weighted approximation by Bernstein-type operators, best algebraic and trigonometric approximation, and L-spline approximation.

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p-Adic and adelic gravity and cosmology

Since 1987, there has been a large activity in application of p-adic and adelic methods in modeling of some physical systems and some other ones, which is known as *p-adic mathematical physics* [1]. In physics, in particular, investigation has been oriented towards string theory, geometry of space-time at the Planck scale, quantum theory, gravity and cosmology.

Adelic approach enables to consider real and p-adic aspects of a system simultaneously. It gives possibility to connect p-adic effects with the real ones. It is a more complete approach than the usual one, which takes into account only real effects. As a result in adelic quantum cosmology one obtains discreteness of space and time at the Planck scale.

In this contribution, it will be presented an introduction to p-adic and adelic analysis, a brief review of p-adic and adelic gravity [2], and p-adic and adelic cosmology [3, 4], including adelic quantum cosmology [5]. It will be also discussed possible p-adic origin of dark matter and dark energy, and their role in evolution of the Universe.

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Dissipativity for mixed order systems

We consider elliptic systems of Douglis-Nirenberg type, defined in mixed order Sobolev spaces of L^p type, endowed with a suitable norm. For such function spaces, we determine the subdifferential of the norm and present conditions on the mixed order elliptic operator to generate a contraction semigroup in the function space.

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Weighted Young's inequalities for radial functions and related embedding theorems

We will present results on convolution inequalities obtained in [1] for Lebesgue and Lorentz spaces with power weights when the functions involved are assumed to be radially symmetric, improving the range of exponents obtained in the general case by R. Kerman in the classical paper [3].

We will then comment on applications of those results to weighted embeddings theorems for radial Besov and Triebel-Lizorkin spaces, in the spirit of those obtained in [2, 4] in the unweighted case, that are the subject of ongoing research.

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Ateb transform as the generalization of Fourier transform

This paper proposes usage of Ateb-functions for protection of information in computer networks. For this purpose, the Ateb- transform based on Ateb-sinus $ca(m, n, x)$ and Ateb-cosine $sa(n, m, x)$, as a generalization of orthogonal Fourier transform, where m, n- parameters of Ateb-function, x-argument, was considered. It was proved that this transform satisfy the properties of linearity, symmetry and similarity. The product and convolution of Ateb-transform and the derivative formula of this transform were deducted. For the case $m = 1$ and $n = 1$ the introduced Ateb-transform shall be known as orthogonal Fourier transform [1]. The function $casa(m, n, x) = ca(m, n, x)^m + sa(n, m, x)^n$ was considered, and the formulas for the direct and inverse Hartley Ateb-transforms were formed. The Hartley Ateb-transform is a real linear operator, and is symmetric and self-inverse properties for Hartley Ateb-transform were proved. It follows that the Hartley Ateb-transform is an unitary and an orthogonal operator.

Discrete transforms shall be used for information security in the computer networks. Therefore, the one-dimensional discrete and two-dimensional discrete Ateb transforms or the one-dimensional discrete and two-dimensional discrete Hartley Ateb-transform were put forward for consideration.

The algorithm of embedding a digital water mark into the image was implemented. Formulas of discrete Ateb-transforms shall be applied to the image. For the transformed image, the well-known additive algorithm of embedding the digital watermark in the frequency domain shall be applied [2]. The algorithm of inverse discrete Ateb transform shall be applied to the image and the presence of a digital watermark shall be verified. A series of experiments in regard to using the processed scheme of embedding the digital watermark in order to study its robustness were performed. The proposed method was tested on image files, but it can also be used to protect audio and video files, as well as electronic text documents in computer networks.

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Functional equations and stress tensor in composite material with circular inclusions

Two-dimensional elastic problem on the plane with mutually disjoint circular inclusions is reduced to functional-differential equations for analytic functions $\phi(z)$ and $\psi(z)$ from the Kolosov-Muskhelishvili representations [1]

$$\sigma_x + \sigma_y = 4\Re\{\phi'(z)\} = 2\left[\phi'(z) + \overline{\phi'(z)}\right],$$

$$\sigma_x - \sigma_y + 2i\tau_{xy} = -2\left[z\overline{\phi''(z)} + \overline{\psi'(z)}\right],$$

$$2\mu(u + iv) = \kappa\phi(z) - z\overline{\phi'(z)} - \overline{\psi(z)}.$$

The functional–differential equations [2] can be solved by the method of successive approximations.

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Spectral Theory of the Hermite Operator on $L^p(\mathbb{R}^n)$

We prove that the minimal operator and the maximal operator of the Hermite operator are the same on $L^p(\mathbb{R}^n)$, $1 < p < \infty$. The domain and the spectrum of the minimal operator (= the maximal operator) of the Hermite operator on $L^p(\mathbb{R}^n)$, $1 < p < \infty$, are computed.

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On effective conductivity formula for bounded symmetric composite material

It is considered effective conductivity of a 2D bounded symmetric composite material [1]. The main method of the study is the reduction of the corresponding boundary value problem to the vector-matrix problem for analytic method. It allows to reduce the problem for a compound domain to the problem for multiply connected circular domain which is solved by the functional equation method [2]. It is obtained an exact formula for effective conductivity.

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RIEMANN BOUNDARY VALUE PROBLEMS ON THE POSITIVE REAL AXIS

In this paper, some Riemann boundary value problems on the positive real axis are presented. Firstly, we introduce the concepts of the principal part and the order at the infinity and the zero point for the holomorphic function on the complex plane cut along the positive real axis. Then, the behavior of Cauchy type integral on the positive real axis at the infinity and the zero point is discussed. Based on those, the Riemann boundary value problems for sectionally holomorphic functions with the positive real axis as their jump curve are solved. As example, some boundary value problems for matrix valued functions are also discussed, which play very important role in the asymptotic analysis for the orthogonal polynomials on the positive real axis.

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On the existence of solutions of ordinary differential equations in Banach spaces

In this talk we shall give sufficient conditions for the existence of solutions of a first order differential equation in a Banach space. We investigate the Cauchy problem

$$x' = f(t, x), \quad x(t_0) = x_0 \tag{1}$$

where E is a Banach space, B is the ball in E and $f : [0, a] \times B \mapsto E$ is a bounded continuous function. Our considerations are inspirated by the paper [1] concerning the unicity of solutions of the problem (1). We suppose that f satisfies generalized α–Nagumo condition $\lim_{r\to 0^+} \alpha(f((t-r, t+r) \cap [0, a] \times X)) \leq \frac{u'(t)}{u(t)}\omega(\alpha(X))$, where α is the measure of noncompactness, $X \subset B$ and $t \in (0, a)$, for some smooth function $u : [0, a] \to [0, \infty)$ with $u(0) = 0$ and $u'(t) > 0$ a.e. on $[0, a]$ and for some continuous and increasing function $\omega : [0, K] \to [0, \infty)$ which is null in 0 and positive every else, and also satisfies the integral inequality $\int_0^r \frac{\omega(s)}{s} ds \leq r, \quad r \in (0, K] \ \ (K > 0)$. Moreover, assuming $\lim_{\substack{t\to 0^+ \\ r\to 0^+}} \frac{\alpha(f(t,B(0,r)))}{u'(t)} = 0$, where $B(0, r)$ is the ball with center 0 and radius r, we shall prove that there exists a compact subinterval J of $[0, a]$ such that the problem (1) has at least one solution defined on J. Our assumptions and proofs are expressed in terms of the measure of noncompactness.

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Total nonnegativity of infinite Hurwitz matrices of entire and meromorphic functions

We fully describe functions generating the infinite totally nonnegative Hurwitz matrices. In particular, we generalize the well-known result by Asner and Kemperman on the total nonnegativity of the Hurwitz matrices of real stable polynomials (see [2, 4]). A criterion alternative to [1] for entire functions to generate a Pólya frequency sequence is also obtained.

The results are based on a connection between a special factorization of totally nonnegative matrices of the Hurwitz type and the expansion of Stieltjes meromorphic functions into Stieltjes continued fractions (regular C-fractions with positive coefficients). An analogous approach was successfully applied earlier in [3] to the case of rational functions.

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On ellipticity of pseudodifferential equations with small parameter

One of the most often used approaches to elliptical pseudodifferential equations(ΨDO) depending on a small parameter is based on Poincaré's asymptotic expansion. Upon this the symbols $a(x, \xi, \epsilon)$ are regarded as continuous function with respect to $\epsilon \in [0, \epsilon_0)$ with the reduced symbol $a_0(x, \xi) = a(x, \xi, \epsilon)\|_{\epsilon=0}$ and define ΨDO $A_\epsilon : H_1 \to H_2$. Further, the problems of convergence of perturbed solutions to the degenerated one in H_1 are considered.

With another approach the principal symbol $a(x, \xi, \epsilon)$ is expanded algebraically to perturbed and non-perturbed terms and it was developed by L. Frank and G. Grubb. Due to these ΨDO algebras some known results for PDE with were generalized to pseudodifferential case.

Both methods allow to obtain asymptotic solutions for a vast amount of physical and mechanical problems. However, there are differential problems with cuspidal boundary where the classical points of view are not applied (e.g. E. Dyachenko and N. Tarkhanov, *Degeneration of Boundary Layer at Singular Points*, preprint 2012). In this case the small variable ϵ can be considered as a new variable and $a(x, \xi, \epsilon)$ is interpreted as an operator-valued symbol transforming $(x, \xi) \in T^*X$ into functions of ϵ. Here we propose the ways to construct these algebras and consider possible applications.

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Diffusion phenomena for the wave equation with structural damping in the $L^p - L^q$ framework

The goal of this talk is to explain the *diffusion phenomena* for the wave equation with structural damping

(1) $$u_{tt} - \triangle u + 2a(-\triangle)^\sigma u_t = 0\,, \qquad u(0,x) = u_0(x)\,, \quad u_t(0,x) = u_1(x)\,,$$

with $a > 0$ and $\sigma \in (0, 1/2)$. We show that u has a heat-type profile for low frequencies, i.e., u behaves like the solution v to

$$v_t + \frac{1}{2a}\,(-\triangle)^{1-\sigma} v = 0\,, \qquad v(0,x) = v_0(x)\,,$$

for suitable choice of initial data v_0. More precisely, we derive $L^p - L^q$ decay estimates for the difference $u - v$ and its time and space derivatives, where $1 \le p \le q \le \infty$, possibly not on the conjugate line, satisfying some additional condition related to σ. In particular, we show that, under suitable assumptions on p, q, σ, a *double diffusion phenomenon* appears, that is, the difference $u - v$ behaves like the solution to

$$w_t + 2a\,(-\triangle)^{\sigma} w = 0\,, \qquad w(0,x) = w_0(x)\,,$$

for a suitable choice of initial data w_0.

The motivation for this work was the results obtained in the articles [2], [3] and by a remark done by the first author of the present work [1] about the $L^2 - L^2$ decay estimates for the solution of (1). In [3] the authors got some $L^p - L^q$ estimates for the solution u of (1), with $1 \le p \le q \le \infty$ and $\sigma \in (0,1]$. The limit case $\sigma = 0$ in (1) corresponds to the classical damped wave, for which the *diffusion phenomena* was already obtained(see [2] and the references therein).

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Special functions and Verma modules

Recently, generalisations to arbitrary dimension of the classical Appell sequences in complex analysis have gained new interest, see e.g. [1, 2, 4]. These sequences are usually defined as polynomials sets, containing polynomials which are indexed by a non-negative integer referring to the degree of homogeneity, on which one may define the action a raising and lowering operator. Under this action, the Appell sequence behaves like a family of generalised monomials, turning the set into a representation for the Heisenberg algebra. In this paper, we construct Appell sequences in higher dimensions by means of infinite-dimensional Verma modules for the Lie algebra $\mathfrak{sl}(2)$, hence obtaining several generalisations of classes of polynomials in terms of bases for Verma modules. This allows a unifying picture for special functions appearing in those frameworks where an underlying dual partner isomorphic to the aforementioned Lie algebra exists (see e.g. [3]). Note that this can also be extended to the Lie superalgebraic refinement, hence obtaining special functions as bases for Verma modules for $\mathfrak{osp}(1,2)$.

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The internal structure of multivariate pairs of dual wavelet frames

We study the multiresolution structure of multi-window wavelet frames. For any nontrivial overcomplete dyadic tight wavelet frame, the induced multiresolution analysis $\{V_j\}_{j\in\mathbb{Z}}$ with associated wavelet spaces $\{W_j\}_{j\in\mathbb{Z}}$ is degenerated, meaning that the standard decomposition $V_1 = V_0 \oplus W_0$ known for wavelet bases turns into $V_1 = W_0$, cf. [1].

We shall extend the latter in 3 ways [2]: First and most significantly, we do not require a tight wavelet frame and verify that the result still holds for a pair of dual wavelet frames. Secondly, we allow for general scaling matrices. Thirdly, the pair of dual wavelet frames is not required to form a frame for $L_2(\mathbb{R}^d)$ but only for a pair of dual Sobolev spaces $(H^s(\mathbb{R}^d); H^{-s}(\mathbb{R}^d)$, cf. [3]. Thus, the dual refinable function does not have to be contained in $L_2(\mathbb{R}^d)$. We also construct pairs of dual wavelet frames for a pair of dual Sobolev spaces from any pair of multivariate refinable functions.

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Inequalities for Angular Derivatives and Boundary Interpolation

The classical Julia–Wolff–Carathéodory theorem asserts that the angular derivative of a holomorphic self-mapping of the open unit disk at its boundary fixed point is a positive number. In 1982, Cowen and Pommerenke proved that if a self-mapping has several boundary regular fixed (or contact) points then the angular derivatives at these points are subject to certain inequalities. In this talk we present a unified approach to establish relations between angular derivatives of such functions with a prescribed (possibly, infinite) collection of either mutual contact points or boundary fixed points. This approach yields diverse inequalities improving both classical and more recent results. We apply them to study the Nevanlinna–Pick interpolation problem with boundary data. Our methods lead to fairly explicit formulas for the set of solutions.

The talk is based on a joint work with V. Bolotnikov and D. Shoikhet.

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Cyclic contractions on G-metric Spaces

In this talk, we discuss the concept of cyclic maps and define some special types of cyclic contractions on G-metric spaces. We investigate the existence and uniqueness of fixed points of Banach type cyclic contractions and generalized weak ψ cyclic contractions on G-metric spaces.

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Quaternionic Hyperbolic Function Theory

We consider harmonic functions with respect to the Laplace-Beltrami operator of the Riemannian metric $ds^2 = x_2^{-2k}\left(\sum_{i=0}^2 dx_i^2\right)$ and their function theory in $\mathbb{R}^3$. If the set of quaternions $\mathbb{H}$ is generated by $1, e_1, e_2$ and $e_{12} = e_1e_2$ satisfying the relation $e_ie_j + e_je_i = -2\delta_{ij}$, $i, j = 1, 2, 12$ and $\mathbb{C}$ is identified by the set $\{x_0 + x_1e_1 | x_0, x_1 \in \mathbb{R}\}$ the modified Dirac operator is introduced by $M_kf = Df + kx_2^{-1}\overline{Qf}$, where Qf is given by the decomposition $f(x) = Pf(x) + Qf(x)e_2$ with $Pf(x)$ and $Qf(x)$ in $\mathbb{C}$ and $\overline{Qf}$ is the usual complex conjugation. Leutwiler noticed around 1990 that if the usual Euclidean metric is changed to a hyperbolic one, that is $k = 1$, then the power function $(x_0 + x_1e_1 + x_2e_2)^n$, calculated using quaternions, is the conjugate gradient of the a hyperbolic harmonic function. We study generalized holomorphic functions, called $k-$hypermonogenic functions satisfying $M_kf = 0$. Note that 0-hypermonogenic are monogenic and 1-hypermonogenic functions are hypermonogenic defined by H. Leutwiler and the author. The function $|x|^{k-1}x^{-1}$ is k-hypermonogenic.

We prove the Cauchy type integral formulas for k-hypermonogenic where the kernels are calculted using the hyperbolic distance of the Poincare upper half space model. Earlier these results have been proved for hypermonogenic functions.

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Ordinary differential equations in algebras of generalized functions

A local existence and uniqueness theorem for ODEs in the special algebra of generalized functions is established, as well as versions including parameters and dependence on initial values in the generalized sense. Finally, a Frobenius theorem is proved.

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Infinity and rest point

In [1, p. 222] there is a description of Alexandroff's one-point compactification of a locally compact non-compact space. In [2, pp. 162-164] it is shown that owing to the Riemann sphere the process of such compactification of a locally compact Hausdorff space can be visualized. The visualization is used in order to construct analytically the homeomorphism of R^2 on $S \setminus \{N\}$. As in [2], $S \setminus \{N\}$ is a sphere minus its north pole N. It is considered the question. To what conditions at infinity (∞) must satisfy an autonomous system of differential equations defined on R^2 in order corresponding to it under the homeomorphism system defined on $S \setminus \{N\}$ had rest point N. Here ∞ and N are considered as "ideal points" of one-point compactifications for R^2 and $S \setminus \{N\}$. The same question is considered for R and part of R.

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Initial-boundary value problems for Zakharov–Kuznetsov equation on the plane

Initial-boundary value problems for Zakharov–Kuznetsov equation

$$u_t + u_{xxx} + u_{xyy} + uu_x = 0 \tag{1}$$

in various domains are considered and problems of well-posedness are studied. The most recent results are related to the problem in a layer $\Pi_T = (0,T) \times \Sigma$, where $\Sigma = \{(x,y) : x \in \mathbb{R}, 0 < y < L\}$ is a horizontal strip of a given width L and $T > 0$ – arbitrary. Initial condition

$$u(0,x,y) = u_0(x,y), \qquad (x,y) \in \Sigma, \tag{2}$$

and Dirichlet boundary condition

$$u(t,x,0) = u(t,x,L) = 0, \qquad (t,x) \in (0,T) \times \mathbb{R}, \tag{3}$$

are set.

For any $\alpha \geq 0$ introduce function spaces

$$L_2^\alpha = H^{0,\alpha} = \{\varphi \in L_2(\Sigma) : (1+x_+)^\alpha \varphi \in L_2(\Sigma)\},$$
$$H^{1,\alpha} = \{\varphi \in H^1(\Sigma) : \varphi, \varphi_x, \varphi_y \in L_2^\alpha\}$$

with natural norms (here $x_+ = \max(x,0)$). Solutions to the considered problem are constructed in spaces $X^{k,\alpha}(\Pi_T)$, $k=0$ or 1, consisting of functions $u(t,x,y)$ such that

$$u \in C_w([0,T]; H^{k,\alpha}), \qquad \sup_{x_0 \in \mathbb{R}} \int_0^T \int_{x_0}^{x_0+1} \int_0^L |D^{k+1}u|^2 \, dydxdt < \infty$$

and if $\alpha > 0$ then, in addition,

$$(1+x)^{\alpha-1/2}|D^{k+1}u| \in L_2\big((0,T) \times (0,+\infty) \times (0,L)\big),$$

where $|D^k\varphi| = \Big(\sum\limits_{k_1+k_2=k} (\partial_x^{k_1}\partial_y^{k_2}\varphi)^2\Big)^{1/2}$.

Theorem 1. *Let $u_0 \in L_2^\alpha$ for a certain $\alpha \geq 0$. Then there exists a weak solution to problem (1)–(3) in the space $X^{0,\alpha}(\Pi_T)$.*

Theorem 2. *Let $u_0 \in H^{1,\alpha}$ for a certain $\alpha \geq 0$ and $u_0|_{y=0} = u_0|_{y=L} = 0$. Let $u_0 \in L_2^\alpha$ for a certain $\alpha \geq 0$. Then there exists a weak solution to problem (1)–(3) in the space $X^{1,\alpha}(\Pi_T)$. If $\alpha \geq 1/2$ such a solution is unique.*

Similar rusults are established for other types of boundary conditions (Neumann and perodic).

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Continuous wavelet transform over an n-dimensional ball

It is well-known that the proper Lorentz group $SO_0(1,n)$ (and also its double covering Spin$^+$(1,n)) is the conformal group of the unit sphere and of the unit ball in $\mathbb{R}^n$. This group gives a well-established theory for the continuous wavelet transform on the unit sphere as studied by the author in [2]. However, for the case of the ball the proper Lorentz group $SO_0(1,n)$ is not sufficient for a wavelet theory, since dilations are not contained in it.

Recently it was developed the continuous wavelet transform on the upper sheet of the 2-hyperboloid [1] by defining a class of suitable dilations on the hyperboloid through conic projection and by incorporating hyperbolic motions belonging to the proper Lorentz group $SO_0(1,2)$. The resulting wavelet transform is invertible whenever the wavelet mother satisfies a particular admissibility condition, which turns out to be a zero-mean condition.

In this talk we will show how to obtain similar results in a ball of $\mathbb{R}^n$ with arbitrary radius. First we define a class of radial relativistic dilation operators for the ball which allow us to define the continuous wavelet transform together with motions on the ball. We study the admissibility condition for this transform in order to have admissible wavelets and we give examples of admissible hyperbolic wavelets. For large radiuses of the ball the continuous wavelet transform on the ball matches the usual continuous wavelet transform in $\mathbb{R}^n$.

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Multiple orthogonal polynomials and their properties

I shall talk about multiple orthogonal polynomials and their properties and connection to Riemann-Hilbert problems and differential equations based on the paper [1]. This is a joint work with W. Van Assche and L. Zhang (KULeuven, Belgium).

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Boundary Problems in the Fracture Mechanics of Solids with Cracks

Solution methods of 2D boundary value problems for solids with multiple fractures of sufficiently arbitrary configurations are developed. For this purposes complex analysis approaches and the technique of singular integral equations are used [1]. As an example, it is considered a fractured anisotropic half-plain loaded by concentrated forces along the straight-line boundary. Fractures are statistically distributed by Gaussian low. It is supposed that the banks of cracks are free from loadings or under the normal pressure. The appropriate boundary problem is reduced to a singular integral equation system that is uniquely solvable due to additional conditions of fracture closure. Further an asymptotic analysis of solutions is conducted and based on such procedure stress intensity factors at the tip of each fracture are calculated. Numerical procedures have been performed in the following sequence. The centers of cracks were got out in a random way on two-dimensional normal distribution with the center in a point z_0. The form of cracks is the polynomials which power got out in a random way from previously defined set. Length and rotation angle of cracks were also set randomly. Being crossed cracks were excluded from consideration. Thus, in the set area rather large number of cracks was distributed. After that numerical calculation of the stress intensity factor at the tip of each crack was carried out. The described procedure was repeated rather large number of times. As a result, the expected value and the variance of stress intensity factors [2] at crack tips have been received.

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Pseudo-Differential Operators on Lie groups

Pseudo-differential operators (PDO's) are primarily defined in the familiar setting of the Euclidean space. In this short talk, I will present recent results regarding PDO's in the settings of Lie groups. This is a joint work with Michael Ruzhansky (Imperial College London).

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Kähler metrics and geometric quantization

Kähler metrics on a compact toric manifold were explicitly parametrized in [1]. We consider natural degenerations of these metrics and their relation with geometric quantization of the toric manifold, [2].

In particular, these degenerations allow to interpolate between geometric quantizations in the holomorphic and real polarizations and show that sections of the prequantum bundle converge to Dirac delta distributions supported on Bohr-Sommerfeld fibers.

Analogous descriptions of Kähler metrics on spherical manifolds and corresponding degenerations will also be considered.

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The Dirichlet problem on the sphere for the modified Cauchy- Riemann operator

In this work we will show how guarantee the existence of the Dirichlet-type boundary value problem solution on the sphere. It was discussed for modified Cauchy-Riemann operator in [4].

$$D_q = \sum_{i=0}^{n} q_i e_i \partial_i$$

where the parameters q_i can be real-valued functions,matrix value function or Clifford Algebra-valued functions in the context of the generalized Clifford Analysis.

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* Trabajo conjunto con

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Redundant Lifting Scheme for Multiscale Keypoint Analysis

A redundant lifting scheme that can build the discrete wavelet transform with translation-invariant property is presented. The original lifting of Sweldens [1, 2] has been widely used in a range of applications, as it can provide a particularly easy way to construct perfect reconstruction filters that are defined even on general domains such as irregular grids over arbitrary surfaces. In particular, any discrete wavelet transform with finite impulse response filters can be decomposed into a finite sequence of simple lifting steps [3]. The computation of the wavelet decomposition or reconstruction implemented via the lifting is efficient and thus fast, because it uses the polyphase decomposition that divides a one-dimensional discrete signal into even and odd components, which is also called decimation or downsampling by a factor of 2. However, due to the nature of the polyphase decomposition, this leads to a large number of artifacts when the signal is reconstructed after modification of its wavelet coefficients.

The proposed redundant lifting scheme does not use the decimation or downsampling. The decomposed output signals are redundant components whose sum has a length of two times from an original signal, but they make the reconstruction robust to the ringing artifact, which can be very important in some applications such as feature detection. Furthermore, the redundancy of the lifting framework can be used for redesigning new wavelet filters. We applied these filters to some applications such as edge detection and keypoint analysis [4] of an image, in order to show that the redundancy of the transforms offers some advantages comparing with the conventional method.

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Gabor transform of analytic functional on the sphere

We have studied analytic functionals on the complex sphere $\tilde{S}(r)$ with radius r measured by the Lie norm in $\mathbf{C}^{n+1}$. The Fourier-Borel transform $\mathcal{F}T$ of an analytic functional T on $\tilde{S}(r)$ is defined by

$$\mathcal{F}T(\zeta) = \langle T_z, \exp(z \cdot \zeta) \rangle,$$

where $z \cdot w = z_1 w_1 + \cdots + z_{n+1} w_{n+1}$ for $z, w \in \mathbf{C}^{n+1}$. We denote by $P_{k,n}(t)$ the Legendre polynomial of degree k and of dimension $n+1$. The extended Legendre polynomial $P_{k,n}(z,w)$ of degree k and dimension $n+1$ is defined by $P_{k,n}(z,w) = (\sqrt{z^2})^k (\sqrt{w^2})^k P_{k,n}(\frac{z}{\sqrt{z^2}} \cdot \frac{w}{\sqrt{w^2}})$, where $z^2 = z \cdot z$. For the analytic functional T, we call $\tilde{S}_k(w) = N(k,n)\overline{\langle T_z, P_{k,n}(z, \overline{w}) \rangle}$ the k-spherical harmonic component of T, where $N(k,n)$ is the dimension of the space of homogeneous harmonic polynomials of degree k in $\mathbf{C}^{n+1}$. Since $\exp(z \cdot \zeta)$ can be expanded by using the extended Legendre polynomials and the entire Bessel functions, $\mathcal{F}T$ can be expressed by using the k-spherical harmonic component of T and the entire Bessel functions.

In this talk, following our previous results we consider the Gabor transform $\mathcal{G}_{\omega_0}T$ of T defined by

$$\mathcal{G}_{\omega_0}T(\tau, a) = \left\langle T_x\, ,\, a^{-(n+1)/2} \exp(-i\omega_0 \cdot \frac{x-\tau}{a}) \exp\left(-\frac{1}{2} \left(\frac{x-\tau}{a} \right)^2 \right) \right\rangle,$$

where $\omega_0 \in \mathbf{R}^{n+1}$ is fixed, $a \in \mathbf{R}_+$ and $\tau \in \mathbf{C}^{n+1}$.

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On the interpolation of orthonormal wavelets with compact support

Daubechies wavelets have good properties; the compact support and the vanishing moment, and often are applied to the numerical analysis, e.g., image compression. They are constructed from the square of their low-pass filter by using the spectral factorization (see [1]). Let m_0^N be the low-pass filter associated with the N-th order Daubechies wavelet and $M_0^N(\xi) = \left| m_0^N(\xi) \right|^2$. Then, M_0^N is given as

$$M_0^N(\xi) = \left(\frac{1+e^{-i\xi}}{2} \right)^{2N} L(\xi),$$

where $L(\xi) = P_N\left(\sin^2 \xi/2\right)$ with

$$P_N(y) = \sum_{k=0}^{N-1} \binom{N-1+k}{k} y^k.$$

Daubechies wavelets correspond to the spectral factorization which yields the minimum phase filter m_0^N. Other choices allow us to get more smoother wavelets, or more symmetric wavelets (e.g., symlets). The N-th order Daubechies wavelet has N vanishing moments. Daubechies [3] constructed wavelets such that scaling functions also have vanishing moments and named them coiflet. The regularities of Daubechies wavelets and coiflets increase with increasing the order of them (see [2], [4], etc.).

In this talk, we introduce a new scheme which constructs new orthonormal wavelets. In some cases, the resulting wavelets are in agreement with our recent works [5] and [6]. Moreover, by combining the scheme with the spectral factorization, we can interpolate between two compactly-supported wavelets. Finally, we report some properties of new wavelets.

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The Wintner–Perko termination principle and new bifurcational geometric methods for the global qualitative analysis of polynomial dynamical systems

We carry out the global qualitative analysis of polynomial dynamical systems. To control all of their limit cycle bifurcations, especially, bifurcations of multiple limit cycles, it is necessary to know the properties and combine the effects of all of their rotation parameters. It can be done by means of the development of new bifurcational geometric methods based on the well-known Weierstrass preparation theorem and the Perko planar termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity) [1]. This principle is a consequence of the principle of natural termination which was stated for higher-dimensional dynamical systems by A. Wintner who studied one-parameter families of periodic orbits of the restricted three-body problem and used Puiseux series to show that in the analytic case any one-parameter family of periodic orbits can be uniquely continued through any bifurcation except a period-doubling bifurcation. The Wintner–Perko termination principle can be applied for studying multiple limit cycle bifurcations of planar polynomial dynamical systems [1].

If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Applying this method, we have solved, e. g., Smale's Thirteenth Problem proving that the Liénard system with a polynomial of degree $2k+1$ can have at most k limit cycles. Generalizing the obtained results, we have also solved the problem of the maximum number of limit cycles surrounding a singular point for an arbitrary polynomial system and Hilbert's Sixteenth Problem for a general Liénard polynomial system with an arbitrary (but finite) number of singular points [2].

Finally, applying the same approach, we consider three-dimensional polynomial dynamical systems and, in particular, complete the strange attractor bifurcation scenario in the classical Lorenz system globally connecting the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles.

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Mathematical modeling of the acute phase of HIV-1 infection

Mathematical models describing relationship between infectious pathogens and an immune system are used for both identification of the model unknown parameters and for comparison and investigation of the various hypotheses, as well as for evaluation in real time of the drugs action and their side effects during a long period of time (years), for investigation of viral genetic variability.

The events during the acute phase of HIV-1 are the important factors for predicting the further course of the infection process. They allow estimation of the time duration of infection latent phase before progression of HIV into AIDS as well as the severity of disease course. Key factors are the viral set point and the T-cell level at the moment of acute phase completion.

The paper provides an overview of a number of publications, beginning with the early models, and to the works of recent years, in which dynamics of the HIV-1 acute phase is described in terms of ordinary differential equations. Most of the modern mathematical models are based on the earlier basic model (see, for example [1]) binding three main compartments: noninfected target T-cells; infected T-cells; free virions. On the base of the comparative analysis the common and individual factors of the mathematical models were determined.

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Blow-up for some differential inequalities

Catastrophes occur in many natural and technical processes. An effective tool for their study is the theory of blow-up of solutions to nonlinear differential equations and inequalities. So far it was developed mostly for differential operators with regular coefficients or for those with singularities on bounded sets, such as a single point [1].

We establish sufficient conditions for blow-up of solutions to nonlinear partial differential inequalities such that their coefficients and/or initial data have singularities on unbounded sets. Our proofs are based on an appropriate modification of the method of nonlinear capacity [2].

An example of our results is as follows. Let $k \in \mathbb{N}$, $q > 1$, $\alpha, \lambda, \mu \in \mathbb{R}$, $T > 0$, and $c > 0$. Denote $\Pi_n = \{(x_1, \ldots, x_n) \in \mathbb{R}^n : x_n = 0\}$. Consider the nonlinear parabolic Cauchy problem of higher order

$$(1) \qquad \begin{cases} u_t - \Delta^k u \geq u^q |x_n|^\alpha & (x \in \mathbb{R}^n \setminus \Pi_n,\ t \in (0,T)), \\ u(x,t) \geq 0 & (x \in \mathbb{R}^n \setminus \Pi_n,\ t \in (0,T)), \\ u(x,0) = u_0(x) \geq c|x|^\lambda |x_n|^\mu & (x \in \mathbb{R}^n \setminus \Pi_n). \end{cases}$$

Theorem. *Let*

$$\alpha > \max\{(\mu - \lambda)(q-1) + 2k, 0\}.$$

Then any solution of the Cauchy problem (1) *blows up.*

Similar results were obtained for a large class of nonlinear elliptic and parabolic inequalities with various geometrical structure of singularities, as well as for systems of such inequalities.

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Computing the time-frequency solution of a differential equation with the generalized input/output method

Differential equations are the most common model for deterministic and random physical phenomena. In general, when the forcing term of the differential equation is nonstationary the solution is nonstationary, and its frequency content changes with time. Time-frequency analysis provides an effective representation of such time-varying spectrum. We discuss the generalized input/output method, which allows the calculation of the time-frequency distribution of the solution to the equation when the forcing term is nonstationary [1]. The method takes advantage of a technique to transform the differential equation in time to a differential equation in the time-frequency domain of the Wigner distribution [2], [3]. The forcing term and solution of the corresponding time-frequency equation are the time-frequency distributions of the forcing term and solution of the time equation, respectively. Although the Wigner distribution is nonlinear, the time-frequency equation is still linear. By using this property, we expand the time-frequency forcing term in a sum of functions, the generalized inputs, and we compute the corresponding time-frequency solutions, the generalized outputs. The linear combination of the generalized outputs returns the time-frequency solution for the given nonstationary forcing term. The terms generalized inputs and generalized outputs refer to the fact that these functions are not proper Wigner distributions. By using the generalized input/output method we compute the time-frequency distribution of the solution when the forcing term belongs to a class of common nonstationary signals. All results are exact. Furthermore, we show how to extend the method to approximate the time-frequency distribution of the solution for a wide class of nonstationary forcing terms.

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Microlocal Analysis for Hyperbolic Equations in the Einstein & de Sitter Spacetime

In the talk we consider the waves propagating in the universe modeled by the so-called Einstein & de Sitter cosmological model. The wave equation in the Einstein & de Sitter spacetime is strictly hyperbolic in the domain with positive time, while the coefficients have singularities at time $t = 0$ (the cosmological singularity, the moment of Big Bang). We set initial data on the hyperplane separated from the singularities and investigate asymptotic behavior of the solution as time approaches zero. We give explicit representation formulas and parametrixes of the Cauchy problem in the terms of Fourier integral operators. This allows us to prove rigorously some known physically motivated asymptotics.

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Wavelet approach for the problems of discerning samples obtaining from acoustic measuremens

The experimental data are obtained after the registration of the backscattered signals of the initial ultrasound impulse. Here the problem of discerning data the obtained from different samples arise. That will mean developing the further methods of skin pathology detecting, [1]-[2], or constructing the correct physical model, cf. [3].

Acoustic experiment results the original signal as the discrete dataset which is modified then using the Hilbert transform. The proposed method is based on decomposition of the modified signal representing by analytic function in the upper half-plane using Daubechies 6 and complex Morlet wavelet, [4]-[5].

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GIT stability, symplectic reduction and cosmic strings

In ongoing work with Luis Alvarez-Consul and Oscar Garcia-Prada, we have recently found an interesting relation between the Coupled Kähler–Yang–Mills (CKYM) equations, introduced in [3], and physical equations describing gravitating vortices over a Riemann surface. These vortices, known in the physics literature as cosmic strings (or topological defects) in the Abelian–Higgs model [2], correspond to static solutions of the Einstein–Yang-Mills–Higgs equations in 4 dimensions for Abelian gauge group. They model the coupling of gravity with a condensed matter system.

In this talk, we explain how this physical interpretation of the CKYM equations arises via dimensional reduction, from 4 to 2 dimensions, in the product of a Riemann surface X with a 2-sphere. When X is the Riemann sphere, the existence problem for the gravitating vortex equations has been previously studied by Y. Yang in [4], leading to sufficient criterion for the existence of solutions. Relying on Yang's criterion and on the general theory for the CKYM equations developped in [1], we will derive a conjectural explicit description of the moduli space of cosmic strings on $\mathbb{CP}^1$ as a finite-dimensional GIT quotient.

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■ **Ricardo García López** Universitat de Barcelona, Barcelona, Spain,
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Reciprocity laws for analytic functions on curves

We will introduce a functional analytic variant of the notion of locally linearly compact topological vector space (also known as Tate space). We will show how this notion can be used to define commutator symbols which satisfy Weil-type reciprocity laws for algebraic curves, both in the complex and in the p-adic analytic setting.

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L^p microlocal properties for vector weighted pseudodifferential operators with smooth symbols

The authors introduce a class of pseudodifferential operators, whose symbols satisfy completely inhomogeneous estimates at infinity for the derivatives, namely:

$$|\partial_\xi^\alpha \partial_x^\beta a(x,\xi)| \leq c_{\alpha,\beta} m(\xi) \Lambda(\xi)^{-\alpha},$$

where $m(\xi)$ is a suitable positive continuous weight function, which indicates the "order" of the symbol, and $\Lambda(\xi) = (\lambda_1(\xi), \ldots, \lambda_n(\xi))$ is a weight vector.
Continuity properties in suitable weighted Sobolev spaces of L^p type are given and L^p microlocal properties studied.

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■ **Claudia Garetto** Loughborough University, UK, email: c.garetto@lboro.ac.uk,
Strictly and weakly hyperbolic Cauchy problems with non-regular coefficients.

In this survey talk I will describe a Colombeau approach to hyperbolic Cauchy problems with non-regular coefficients. The results presented during the talk have been obtained in collaboration with Michael Oberguggenberger (Innsbruck University) for equations without multiplicities (strictly hyperbolic) and in collaboration with Michael Ruzhansky (Imperial College London) for equations with multiplicities (weakly hyperbolic).

■ **Graziano Gentili** Dipartimento di Matematica e Informatica "U.Dini" - Università di Firenze, email: gentili@math.unifi.it

Results of geometric theory of hypercomplex functions on the unit ball of quaternions

The geometric theory of slice regular functions is developing nicely. In this talk we present a few recent results of geometric function theory in the open unit ball $\mathbb{B}$ of the space $\mathbb{H}$ of quaternions. The quaternionic versions of the Bohr, the Bloch-Landau and the Landau-Toeplitz theorems are illustrated, and their differences with their complex counterparts are put under scrutiny. Questions of rigidity of Burns-Krantz type are also addressed.

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On countinuity of the solution map for the cubic 1d periodic half - wave equation

The talk treats the well – posedness of the semilinear equation

$$(i\partial_t - |D_x|)u = \sigma|u|^2 u \text{ for } t \geq 0 \ , \qquad (1)$$

where $\sigma = \pm 1$ and $u(t, x)$ is assumed to be 2π periodic in x.

For the case $\sigma = -1$ and $x \in \mathbb{R}$ a blow - up result is established in [1].

We study the case $\sigma = +1$ and impose initial data

$$u(0, x) = f(x) \in H^s(0, 2\pi)$$

with $s < 1/2$ by using suitable Bourgain type spaces.

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■ **Pelin Güven Geredeli** Department of Mathematics, Faculty of Science, Hacettepe University, Beytepe 06800 Ankara, TURKEY, email: pguven@hacettepe.edu.tr,

Existence of the global attractors for parabolic equations involving weighted p-Laplacian operator

In this work, we study the long time behaviour (in the sense of attractors) of the quasilinear parabolic equations with variable coefficients and involving weighted p-Laplacian operator. We prove that the dynamical systems have global attractor in $L^2(R^n)$.

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On hyperholomorphic functions of spatial variable and some properties of a Cauchy type integral in quaternion analysis

Let $\mathbb{H}(\mathbb{C})$ be the algebra of complex quaternions $a = \sum_{k=0}^{3} a_k \boldsymbol{i_k}$, where $\{a_k\}_{k=0}^{3} \subset \mathbb{C}$, $\boldsymbol{i}_0 = 1$ be the unit, $\boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3$ be imaginary quaternion units. Let Ω be a domain in $\mathbb{R}^3 \ni z := z_1\boldsymbol{i}_1 + z_2\boldsymbol{i}_2 + z_3\boldsymbol{i}_3$, let function $f : \Omega \to \mathbb{H}(\mathbb{C})$ has first order partial derivatives, and let $D_l[f] := \sum_{k=1}^{3} \boldsymbol{i_k} \frac{\partial f}{\partial z_k}$, $D_r[f] := \sum_{k=1}^{3} \frac{\partial f}{\partial z_k} \boldsymbol{i_k}$.

Definition. *Function $f := f_0 + f_1\boldsymbol{i_1} + f_2\boldsymbol{i_2} + f_3\boldsymbol{i_3}$ is called left-hyperholomorphic or right-hyperholomorphic in a domain Ω when its components f_0, f_1, f_2, f_3 are $\mathbb{R}^3$-differentiable functions in Ω and satisfy the condition $D_l[f] = 0$ or $D_r[f] = 0$ respectively.*

Let $\delta > 0$, $\Gamma_{z,\delta} := \{\zeta\in\Gamma : |\zeta - z| \leqslant \delta\}$, let $\mathrm{mes}\,\Gamma_{z,\delta}$ be the surface measure of the set $\Gamma_{z,\delta}$, and $d(\Gamma_{z,\delta})$ be its diameter. The next theorem is an analog of the Cauchy theorem from complex analysis.

Theorem (see [1])**.** *Let Ω be a bounded domain with piecewise smooth boundary Γ, which for arbitrary $z \in \mathbb{R}^3$, $\delta > 0$ satisfies the condition*

$$\frac{d(\Gamma_{z,\delta})}{\mathrm{mes}\Gamma_{z,\delta}} \leqslant \Lambda,$$

where Λ is a positive constant, let a function $f : \overline{\Omega} \to \mathbb{H}(\mathbb{C})$ be right-hyperholomorphic in Ω and continuous in $\overline{\Omega}$, and let a function $g : \overline{\Omega} \to \mathbb{H}(\mathbb{C})$ be left-hyperholomorphic in Ω and continuous in $\overline{\Omega}$. Then

$$\iint\limits_{\Gamma} f(z)\,\nu(z)\,g(z)\,ds = 0, \tag{1}$$

where $\nu(z)$ is the unit normal vector to the surface Γ in the point z.

The formula (1) was proved formerly (see [2]) under the additional requirement of continuity for partial derivatives of functions f and g.

We also found sufficient conditions for existence of a quaternion singular Cauchy integral on a closed rectifiable regular surface in the space $\mathbb{R}^3$ and proved an upper estimate for its continuity module in terms of the continuity module of the integrand.

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■ **Dorin Ghisa** - York University, Toronto, Canada, email: dghisa@yorku.ca

On the Generalized Riemann Hypothesis

It is known [1] that for any Dirichlet primitive character χ, the corresponding L-function $L(s,\chi)$ verifies a functional equation relating $L(s,\chi)$ and $L(1-s,\overline{\chi})$ such that s_0 is a non trivial zero of $L(s,\chi)$ if and only if $1-\overline{s}_0$ is a non trivial zero of $L(s,\chi)$. Then, proving that this can happen only if $s_0 = 1-\overline{s}_0$, i.e. $\mathrm{Re}\,s_0 = 1/2$ is equivalent to proving the generalized Riemann Hypothesis for this class of functions.

We show that such an equality is a corollary of some global mapping properties [2] of the functions $L(s,\chi)$.

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The singular perturbation problem for Kirchhoff equations

We consider a family of second order Kirchhoff equations

$$\epsilon u_\epsilon''(t) + m(|A^{1/2}u_\epsilon(t)|^2)Au_\epsilon(t) + \frac{1}{(1+t)^p}u_\epsilon'(t) = 0$$

where ϵ is a small parameter, $0 \leq p \leq 1$, A is a non-negative linear operator on a Hilbert space and m is a non-negative continuous function. When $\epsilon \to 0$ we formally obtain the first order equation

$$u'(t) + (1+t)^p m(|A^{1/2}u(t)|^2)Au(t) = 0.$$

We prove optimal decay estimates for the solutions of the hyperbolic problem, and optimal decay-error estimates for the difference between u_ϵ and u.

These estimates show a quite surprising fact: in the nondegenerate case, that this when $m \geq \nu > 0$, the analogy between parabolic equations and dissipative hyperbolic equations is weaker than in the degenerate case ([3]). In the degenerate case, when $m(\sigma) = \sigma^\gamma$, under some assumption on the initial data the difference between solutions decays faster than the two terms separately while under the complementary assumption the optimal decay-error estimates involve a decay rate which is slower than the decay rate of the two terms ([1], [2]).

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Generalized functions as a category of smooth set-theoretical maps

We present a new approach to generalized functions, so-called generalized smooth functions (GSF), as set-theoretical maps defined on, and taking values in $\widetilde{\mathbb{R}}$, the non-Archimedean Colombeau's ring of generalized numbers (CGN). We prove that GSF form a concrete category $\mathcal{GC}^\infty$ which unifies and extends Schwartz distributions and Colombeau generalized functions. The calculus of these generalized functions is closely related to classical analysis, with point values, composition, non linear operations, the usual rules for differentiation and integration and classical theorems like the intermediate value theorem, the mean value theorems, the extreme value theorem and several forms of Taylor formula. The basic idea is to impose the minimal logical conditions to have maps of CGN generated by nets (u_ε) of ordinary smooth functions, with $u_\varepsilon \in \mathcal{C}^\infty(\Omega_\varepsilon, \mathbb{R}^d)$ and $\Omega_\varepsilon \subseteq \mathbb{R}^n$ open. The differential calculus of this type of functions can be developed in an intrinsic way using the Fermat-Reyes theorem (see [1]), which states existence and uniqueness of a generalized smooth incremental ratio r satisfying $f(x+h) = f(x)+h\cdot r(x,h)$. The integral calculus of GSF can be developed proving existence and uniqueness of primitives.

We finally present a useful characterization of Schwartz distributions among GSF that permits to easily prove that functions like a generalized mollifier $f(x) = i^{-\alpha} \cdot \varphi(i^{-1} \cdot x)$, where i is infinitesimal, $\alpha \neq 1$ and $\varphi \in \mathcal{D}(\mathbb{R})$, or like $g = I \cdot \chi_{B_i(q)}$, where $\chi_{B_i(q)}$ is the characteristic function of a ball of infinitesimal radius i and I is an infinite density, or like a wave with infinite frequency $h(t) = (\cos(I \cdot t), \sin(I \cdot t))$, are not distributions.

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On numerical invariants of a Fuchsian system

To a system of Fuchsian differential equations$df = \omega f$ defined on a Riemann surface X of genus g, where ω is a meromorphic 1-form with the set of singular points $S = s_1, s_2, , s_m$, we will associate certain numerical invariants [1]. These invariants are numerical characteristics of algebraic, analytic and topological objects obtained from the system. The above Fuchsian system determines the monodromy representation $\rho : \pi_1(X - S, z_0) \to GL(n, C)$ from which on the non-compact Riemann surface $X - S$ one obtains the holomorphically trivial vector bundle $E_\rho \to X - S$ with the holomorphic connection ∇, whose connection matrix is the 1-form ω. It is known that the bundle (E_ρ, ∇_ρ) may be extended to the whole Riemann surface X so that one obtains a family of holomorphically nontrivial bundles with logarithmic connections, which does not in general contain any semistable bundles. If (E^0, ∇^0) is the canonical extension of the given bundle [2], then using the monodromy representation one can compute the Chern number, Fuchs weight, and the moduli spece of holomorphic deformations of this bundle. We will use these invariants to investigate the s. c. inverse problem (see [1] or [3]). The latter purports construction, from a given representation as above, of a Fuchs system whose monodromy representation coincides with ρ. Along with this problem we will also consider the linear conjugation problem with the piecewise-constant boundary condition which is constructed from the monodromy matrices.

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Optimal decay estimates for semi-linear parabolic and hyperbolic equations

We consider abstract semi-linear parabolic equations of the form

$$u'(t) + Au(t) + f(u(t)) = 0,$$

and abstract semi-linear dissipative hyperbolic equations of the form

$$u''(t) + Au(t) + u'(t) + f(u(t)) = 0,$$

where A is a non-negative linear operator with closed range on a Hilbert space. We prove optimal decay estimates for solutions under suitable assumptions concerning the behavior of the non-linear terms near the origin.

In the case where the linear part has a nontrivial kernel, we show the coexistence of slow solutions (with polynomial decay rate) and fast solutions (with exponential decay rate). We also classify all possible exponential decay rates.

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Refining Schatten class properties of Hankel operators

We will present some results about Schatten class properties of Hankel operators with Hölder continuous symbols on Hardy spaces of strictly pseudo-convex domains in Stein manifolds. In particular, we will discuss some refinements to weak Schatten class properties in the limit case. For the circle, such a refinement is easily carried out and has an interesting consequence for functionals on Hölder spaces.

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On the Convergence of Multiple Fourier series of Functions of Bounded Partial Generalized Variation

The convergence of multiple Fourier series of functions of bounded partial Λ -variation is investigated. The sufficient and neccessary conditions on the sequence $\Lambda = \{\lambda_n\}$ found for the convergence of partial sums of Fourier series of functions of bounded partial Λ -variation. We introduce a new concept of Λ -variation of multivariate functions and investigate its connection with the convergence of double Fourier series.

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Equicontinuity of plane homeomorphisms with controlled p-module

We deal with plane homeomorphisms preserving integrally quasiinvariant the weighted p-module and provide conditions ensuring the local Hölder continuity of such mappings with respect to euclidian distances and to their logarithms. The related properties, as equicontinuity and normality, are also discussed.

Let G be a domain in $\mathbb{C}$ and $Q : G \to [0, \infty]$ be a Lebesgue measurable function. A homeomorphic mapping $w = f(z) : G \to \mathbb{C}$ is called *Q-homeomorphism* with respect to p-module, if

$$\mathcal{M}_p(f\Gamma) \le \int\limits_G Q(z)\rho^2(z)dm(z) \tag{1}$$

for every family Γ of curves located in G and any ρ admissible for Γ.

The following result states the Hölder continuity of Q-homemorphisms with respect to p-module

Theorem 1. *Let G and G^* be domains in $\mathbb{C}$, and let $f : G \to G^*$ be a Q-homeomorphism with respect to p-module, $1 < p < 2$, with $Q(z) \in L^\alpha(G)$, $\alpha > \frac{2}{2-p}$. Then for an arbitrary compact set $F \subset G$ and for any pair of points $z, \zeta \in F$, such that $|z - \zeta| < \delta$, $\delta = \frac{1}{4}\mathrm{dist}\,(F, \partial G)$, the following inequality holds*

$$|f(z) - f(\zeta)| \le \lambda_p \|Q\|_\alpha^{\frac{1}{2-p}} |z - \zeta|^{1-\frac{2}{\alpha(2-p)}}, \tag{2}$$

with a constant λ_p depending only on p.

The above estimate (2) can be refined by exchanging the euclidian distance by its logarithmic counterpart.

Theorem 2. *Let G and G^* be two domains in $\mathbb{C}$, $\zeta \in G$, and*

$$Q \in L^{\frac{2}{2-p}}(B(\zeta, r_0)), \qquad r_0 \le \min\left(1, \mathrm{dist}^4(\zeta, \partial \mathrm{G})\right).$$

Then for every Q-homeomorphism $f : G \to G^*$ with respect to p-module, $1 < p < 2$,

$$|f(z) - f(\zeta)| \left(\log \frac{1}{|z - \zeta|}\right)^{\frac{p}{2(2-p)}} \le C_p \|Q\|_{\frac{2}{2-p}}^{\frac{1}{2-p}}, \qquad |z - \zeta| < r_0, \tag{3}$$

where $\|Q\|_{\frac{2}{2-p}} = \left(\int\limits_{B(\zeta, r_0)} Q^{\frac{2}{2-p}}(z)\, dm(z)\right)^{\frac{2-p}{2}}$ *and $C_p > 0$ is a constant depending only on p.*

The sharpness with respect to the logarithm order $p/2(2-p)$ in the estimate (3) is illustrated by such automorphism of the unit disk

$$w = e^{i\theta}\left(1 + \frac{2-p}{p-1}\log\frac{1}{|z|}\right)^{-\frac{p-1}{2-p}}, \quad z \ne 0, \quad \text{and} \quad w(0) = 0, \quad z = |z|e^{i\theta}.$$

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Conformal Weights on Sobolev Soaces

Let Ω be a simply connected plane domain with nonempty boundary. We study embeddings of Sobolev spaces $\overset{\circ}{W}{}^1_p(\Omega)$ into weighted Lebesgue spaces $L_q(\Omega, h)$ with a "universal" weight that is the Jacobian of a conformal homeomorphism φ from Ω to the unit disc $B(0, 1)$. Weighted Lebesgue spaces with such weights depend only on the conformal (hyperbolic) structure of the domain Ω. For this reason, we call the weights $h(z)$ conformal weights.

Compactness of embeddings of Sobolev spaces $\overset{\circ}{W}{}^1_2(\Omega)$ into $L_q(\Omega, h)$ is proved for any $1 \leq q < \infty$. With the help of Brennan's Conjecture, we extend these results to the Sobolev spaces $\overset{\circ}{W}{}^1_p(\Omega)$. In this case q depends on p. Applications to degenerate elliptic boundary value problems will be discussed.

We use a version of Brennan's Conjecture for composition operators on Sobolev spaces proposed recently by the authors [2].

The novel points include the use of a "transfer" scheme of Sobolev type embedding theorems from regular domains to non-regular domains (proposed in [1]) in combination with the Riemann Mapping Theorem and the Brennan's Conjecture. The "transfer" scheme is based on systematic applications of the theory of composition operators on Sobolev spaces.

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Radial positive definite functions and best approximation in $L^2(\mathbb{R}^n)$

Let K^n be a class of radial positive definite functions $f\colon \mathbb{R}^n \to \mathbb{R}$ of spherical exponential type 2, $\|f\|_1 < \infty$, $\widehat{f}(0) = 0$,

$$F_r(x, f) = \sum_{k=1}^{r} (-1)^{k-1} \frac{\binom{2r}{r-k}}{\binom{2r}{r-1}} f(kx), \quad r \in \mathbb{N},$$

and

$$\lambda_r^n = \inf\{\lambda\colon F_r(x, f) \leq 0,\ |x| \geq \lambda,\ f \in K^n\}.$$

Lemma 1 ([1]). *The exact Jackson–Chernykh inequality*

$$E_\sigma(f)_2 \leq \binom{2r}{r}^{-1/2} \omega_r\left(\frac{\delta^*}{\sigma}, f\right)_2, \quad \delta^* = 2\lambda_r^n$$

is valid for any function $f \in L^2(\mathbb{R}^n)$. This inequality fails for any argument $\delta < \delta^*$.

Theorem 2 ([2]). *We have $\lambda_1^n = q_\alpha$. The unique extremal function is*

$$f_1^*(x) = \frac{j_\alpha^2(|x|)}{q_\alpha^2 - |x|^2}, \quad \alpha = n/2 - 1.$$

Here j_α is normalized Bessel function, q_α is it's first zero.

In particular cases theorem 2 was proved by N. Chernykh (the case of torus $\mathbb{T}$), B. Logan ($n = 1$), and A. Moskovskiy ($n = 3$).

Theorem 3 ([1]). *We have* $\lambda_1^3 = \lambda_2^3 = \pi$.

Theorem 4. *We have* $q_\alpha \leq \lambda_r^n \leq q_{\alpha+1}$. *The following function provides the upper bound*

$$f(x) = \frac{j_\alpha^2(|x|)}{q_\alpha^2 - |x|^2} + \frac{(j_\alpha')^2(|x|)}{q_{\alpha+1}^2 - |x|^2}.$$

For $n = 1$ in the case of torus theorem 4 was proved by N. Chernykh.

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Gevrey class regularity for a strongly damped wave equation with hyperbolic dynamic boundary conditions

We consider a linear system of PDEs of the form

$$\begin{aligned} u_{tt} - c\Delta u_t - \Delta u &= 0 \quad \text{in} \quad \Omega \times (0,T) \\ u_{tt} + \partial_n(u + cu_t) - \Delta_\Gamma(c\alpha u_t + u) &= 0 \quad \text{on} \quad \Gamma_1 \times (0,T) \\ u &= 0 \quad \text{on} \quad \Gamma_0 \times (0,T) \\ (u(0), u_t(0), u|_{\Gamma_1}(0), u_t|_{\Gamma_1}(0)) &\in \mathcal{H} \end{aligned}$$

on a bounded domain Ω with boundary $\Gamma = \Gamma_1 \cup \Gamma_0$. We show that the system generates a strongly continuous semigroup $T(t)$ which is analytic for $\alpha > 0$ and of Gevrey class for $\alpha = 0$. In both cases the flow exhibits a regularizing effect on the data. In particular, we prove quantitative time-smoothing estimates of the form $\|(d/dt)T(t)\| \lesssim |t|^{-1}$ for $\alpha > 0$, $\|(d/dt)T(t)\| \lesssim |t|^{-2}$ for $\alpha = 0$. Moreover, when $\alpha = 0$ we prove a novel result which shows that these estimates hold under relatively bounded perturbations up to 1/2 power of the generator.

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Cauchy problems for hyperbolic systems with characteristics admitting superlinear growth for $|x| \to \infty$

We study the Cauchy problem for classes of first order hyperbolic systems $D_t + A(t, x, D_x)$ for $t \in \mathbb{R}$, $x \in \mathbb{R}^n$, where $A(t, x, D_x) = \{A_{jk}(t, x, \xi)\}_{j,k=1}^m$ is first order $m \times m$ matrix valued pseudodifferential operators in x depending continuous in $t \in \mathbb{R}$. We consider real characteristic roots admitting superlinear growth with respect to $|x| \to \infty$.

We outline some new results on well–posedeness in weighted function spaces for classes of systems with principal parts first order differential operators with characteristic admitting arbitrary superlinear growth on infinity under Hamiltonian integrability conditions on vector fields in $\mathbb{R}^n$. We discuss also applications to the action of Fourier integral operators in weighted function spaces.

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Using the Schottky-Klein prime function to solve free boundary problems in multiply connected domains

The Schottky-Klein prime function is a special transcendental function which plays a central role in problems involving multiply connected domains. This function can be used to great advantage in many varied applications. In this talk, we will explore two different free boundary problems (arising in fluid mechanics) defined over two distinct multiply connected geometries. For both problems, we will show that it has been expedient to employ the Schottky-Klein prime function and its associated function theory in order to construct analytical solutions. We will also outline a novel numerical method which we have recently derived to rapidly and accurately compute the Schottky-Klein prime function.

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Quaternion functions method for the Moisil-Theodorescu system and it's applications

We used a quaternion function method [1] for the Moisil-Theodorescu system (MTS) [2, 3]. Solutions of the MTS are (left-) regular quaternion functions $f(\mathbf{r}) = f_0(\mathbf{r}) + \mathbf{f}(\mathbf{r}) = f_0(x, y, z) + \mathbf{i}f_x(x, y, z) + \mathbf{j}f_y(x, y, z) + \mathbf{r}f_z(x, y, z)$ of a reduced quaternion variable $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$. The analogues of main theorems of complex analysis for the MTS in quaternion forms are established: Cauchy, Cauchy integral formula, Taylor and Loran series, approximation theorems, Cauchy type integral properties. The analogues of a positive powers (inner spherical monogenics) are investigated: the set of recurrence formulas between inner spherical monogenics and explicit formulas for them are established. Some applications of regular function in elasticity theory and hydrodynamics are given. The generalized Kolosov-Muskhelishvili formulae has obtained in the form:

$$2\mu\mathbf{u}(\mathbf{r}) = \varkappa\Phi(\mathbf{r}) + \overline{\mathbf{r}\varphi(\mathbf{r})} - \overline{\psi(\mathbf{r})},\ \varkappa = 8\nu - 7,\ \Phi = \nabla\varphi,\ \varkappa\Phi_0 = \mathbf{r}\cdot\boldsymbol{\varphi} + \psi_0. \quad (1)$$

By means of the formula (1) in star-shaped domains it is shown, that each main problem of an elastic sphere equilibrium can be reduced to three independent Dirichlet and Neumann problems and solutions of all these problems are expressed in a quadrature with the hypergeometric Appel functions. A quaternion analogue of the Muskhelishvili integral equations, a basis of quaternion boundary elements methods are established. It is shown that some ill-posed problems in elasticity and hydrodynamics are reduced to problems of a regular quaternion function continuation from part of the boundary into the interior of a domain.

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Effective Conductivity of 2D Disk – Ring Composite Material

For 2D bounded composite material geometrically composed by a disk of variable radius r and an outer ring it is determined in an analytic form the x-component of the effective conductivity tensor. Namely, it is shown that the x-component is a sum of geometrical progression with respect to powers of r^2 for all sufficiently small r. If one have to construct the composite material with a prescribed value of the x-component of the effective conductivity tensor and given value of the conductivity of matrix (inclusion), then obtained formula helps to find a functional representation of the conductivity of inclusion (matrix) in terms of r.

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On M-conformal mappings and their geometric properties

Conformal mappings in the plane are described by holomorphic functions. The theory of holomorphic functions can be generalized to higher-dimensional Euclidean spaces. It is well known that the definition of hyperholomorphic functions by means of a generalized Cauchy-Riemann system, certain series expansions or limits of special differences are equivalent. The question is whether hyperholomorphic functions can be defined also by geometrical properties. First ideas can be found in [6], [4] and [1] where the derivability of functions is characterized by means of differential form. More precisely, surface and volume forms must be related in a certain way to define the hypercomplex derivability of a Clifford algebra valued function. Malonek introduced in [3] the concept of M-conformal mappings in a similar way but motivated by geometric properties of the hyperholomorphic functions. In a series of papers, Morais et. al. (see e.g. [2]) studied the class of M-conformal mappings as a subclass of quasi-conformal mappings and found that M-conformal mappings map infinitesimal small balls to special ellipsoids. These ellipsoids are characterized by the property that the length of the longest semi-axis must be the sum of the two other semi-axes. The open question is whether this geometrical property characterizes exactly the class of M-conformal mappings or not. For the null solutions of a Cauchy-Riemann operator with respect to the standard generators of the Clifford algebra this is true only by adding some (strong) conditions. If we extend the class of mappings to the class of ψ-hyperholomorphic functions as they were introduced by Shapiro and Vasilevski in [5] then this question finds a positive answer. This will be shown in the talk and some other properties of these mappings will be discussed.

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On the geometrically nonlinear theory for non-shallow spherical shell

In the present paper, using the method of I. Vekua [1], the three dimensional problems of the non-linear theory of elasticity are reduced to the tow dimensional problems of non-shallow spherical shell [2].

The components of the deformation tensor have the following form:

$$e_{ij} = \frac{1}{2}(\mathbf{R}\partial_i \mathbf{u} + \mathbf{R}_i \partial_j \mathbf{u} + \partial^k \mathbf{u} \partial_k \mathbf{u}),$$

where $\mathbf{R}$ are covariant basis vectors, $\mathbf{u}$ is the displacement vector.

Using the method of the small parameter, approximate solutions of these equations are constructed [3], [4]. The small parameter $\varepsilon = \frac{h}{R}$, where $2h$ is the thickness of the shell, R is the radius of the middle surface of the spherical shell. Some boundary value problems are solved for the approximation of order $N = 0$.

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A coupled coincidence point theorem via implicit relation

In this talk, we discuss the existence of a coupled coincidence point for mappings $F : X \times X \to X$ and $g : X \to X$, where F has the mixed g-monotone property via an implicit relation. The presented results improves and extends various results in the literature. We also consider some examples to illustrate our work.

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The Amalgam Spaces $W\left(L^{p(x)}, l^{\{p_n\}}\right)$ and boundedness of Hardy-Littlewood maximal operators

Let $L^{q(x)}(\mathbb{R})$ be variable exponent Lebesgue space and $\ell^{\{q_n\}}$ be discrete analog of this space. In the second section we define the amalgam spaces $W\left(L^{p(x)}, L^{q(x)}\right)$ and $W\left(L^{p(x)}, \ell^{\{q_n\}}\right)$ and discuss some basic properties of these spaces. Since the global components $L^{q(x)}(\mathbb{R})$ and $\ell^{\{q_n\}}$ are not translation invariant, they are not Wiener amalgam spaces. We show that there are similar properties of these spaces to the Wiener amalgam spaces. We also show that there is a variable exponent $p(x)$ such that $\ell^{\{p_n\}}$ is the discrete space of $L^{p(x)}$ and $W\left(L^{p(x)}, \ell^{\{p_n\}}\right) = L^{p(x)}$. In the third section we study on the frame expansion in $L^{p(x)}(\mathbb{R})$. At the end of this work we prove that the Hardy-Littlewood maximal operator from $W\left(L^{s(x)}, \ell^{\{t_n\}}\right)$ into $W\left(L^{u(x)}, \ell^{\{v_n\}}\right)$ is bounded under some assumptions.

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Optimal decay estimates of the solutions to some second order evolution problems

We report on a joint work with Marina Ghisi and Massimo Gobbino. We consider a class of semi-linear dissipative equations of the form

$$u''(t) + u'(t) + Au(t) + \nabla F(u(t)) = 0$$

where $A = A^* \geq 0$ is a possibly unbounded operator on a real Hilbert space H with non-trivial kernel. Under appropriate assumptions on F, we prove that all solutions decay to 0, as $t \to +\infty$, at least as fast as a suitable negative power of t. Moreover, we establish the existence of a nonempty open set of initial data in $D(A^{1/2}) \times H$ for which the previous decay estimate is optimal. Our results are stated and proved in an abstract Hilbert space setting, and then applied to partial differential equations, the model case being the damped hyperbolic equation "at resonance"

$$u_{tt}(t,x) + u_t(t,x) - \Delta u(t,x) - \lambda_1 u(t,x) + |u(t,x)|^p u(t,x) = 0$$

in $[0,+\infty) \times \Omega$, with homogeneous Dirichlet boundary conditions.

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Bloch periodicity of convolution operators and applications to differential equations

We study the structural properties of spaces of Bloch periodic functions. In particular, we discuss the Bloch periodicity of the convolution $f * g$ of a is Bloch periodic function f and a (Lebesgue) measurable function g satisfying some additional assumptions. Then we make extensive use of the results to examine asymptotically Bloch–periodic solutions to some differential and functional equations as well as a Volterra-like equation.

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Spectra of Functionalized Operators Arising from Hypersurfaces

Functionalized energies, such as the Functionalized Cahn-Hilliard, model phase separation in amphiphilic systems, in which interface production is limited by competition for surfactant phase, which wets the interface. This is in contrast to classical phase-separating energies, such as the Cahn-Hilliard, in which interfacial area is energetically penalized. In binary amphiphilic mixtures interfaces are characterized by bilayers, which divide the domain of the dominant phase, A, via thin layers of phase B formed by homoclinic connections. Evaluating the second variation of the Functionalized energy at a bilayer interface yields a functionalized operator. We characterize the center-unstable spectra of Functionalized operators and obtain resolvent estimates to the operators associated with gradient flows of the Functionalized energies. This is an essential step to a rigorous reduction to a sharp-interface limit.

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Asymptotics in a Complex Frequency Domain and GPR Problems

A ground penetrating radar system consists of a transmitter located over the surface and a receiver located on the surface. A pulsed electromagnetic field is transmitted. The receiver measures the horizontal components of both the electric and magnetic fields. The Ground Penetrating Radars problem is in recovering electrical characteristics of the medium from the readings of the receiver. This problem is ill-posed and cannot be solved precisely because of insufficiency of data. We will discuss approximate solutions of the problem based on asymptotic solutions of Maxwell's equations in the complex frequency domain. To establish the limits of applicability of the method, we derive error estimates and show that the method is numerically efficient if thickness of layers is not less than some resolution threshold (which is of the order of wave length). The efficiency of the method is illustrated by numerical testing.

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Stokes-structures for meromorphic connections

Fundamental results of K. Kedlaya and T. Mochizuki provide deep insight into the structure of meromorphic connections in any dimension. In particular, they enable to extend the notion of Stokes structures from the case of curves to the general situation (see [1]). In the talk, i will present an overview over these results and then focus on applications for period integrals and for questions regarding various functors, e.g. direct images and Fourier transform.

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On second order weakly hyperbolic equations and the ultradifferentiable classes

Let us consider the Cauchy problem for second order weakly hyperbolic equation with time dependent coefficient:

$$\begin{cases} \left(\partial_t^2 - a(t)^2\Delta\right) u = 0, \quad (t,x) \in (0,T] \times \mathbf{R}^n, \\ (u(0,x), u_t(0,x)) = (u_0(x), u_1(x)), \quad x \in \mathbf{R}^n, \end{cases} \tag{1}$$

where $a(t) \in C^\infty([0,T))$ satisfies $C^{-1}\lambda(t) \leq a(t) \leq C\lambda(t)$ with a positive constant $C > 1$ and a strictly decreasing function $\lambda(t)$ satisfying $\lambda(T) = 0$. Moreover, we assume that

$$|a^{(k)}(t)| \leq \lambda(t) M_k \rho(t)^k \quad (k = 1, 2, \ldots)$$

for a positive function $\rho(t)$ and a logarithmical convex sequence $\{M_k\}$. Our main purpose is to describe the weight function μ for the estimate

$$|(\hat{u}(t,\xi), \hat{u}_t(t,\xi))| \leq e^{\mu(\langle\xi\rangle)} |(\hat{u}_0(\xi), \hat{u}_1(\xi))|,$$

which implies the well-posedness of (1) in μ-ultradifferentiable class of Beurling-Roumieu type, by $\lambda(t)$, $\rho(t)$ and $\{M_k\}$.

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Generalized functions and basic symplectic geometry

Following up on the recent development and results on symplectic modules over rings of generalized numbers, we discuss the basic structures of a non-smooth symplectic geometry in terms of a given closed non-degenerate generalized two-form on a smooth manifold. The main focus lies on investigations of a local standard representation in terms of non-smooth Darboux coordinates, generalized Lagrangian submanifolds, and the Poisson structure on generalized functions or the Lie algebra structure of corresponding Hamiltonian vector fields.

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Becker's quasiconformal extension criterion for L^d-Loewner chains

The main interest of this talk is so-called a *Loewner chain*, a parametrized holomorphic univalent function $f_t(z) = e^t z + a_2 z^2 + \cdots$ $(t \geq 0)$ on $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ satisfies the inclusion relationship of the image domain $f_s(\mathbb{D}) \subset f_t(\mathbb{D})$ for all $0 \leq s \leq t < \infty$. It is known that $f_t(z)$ is absolutely continuos on $t \in [0, \infty)$ for each $z \in \mathbb{D}$, and satisfies the partial differential equation

$$\partial_t f_t(z) = z \partial_z f_t(z) p(z, t),$$

where p is a *Herglotz function*, i.e., analytic on $z \in \mathbb{D}$ for each $t \in [0, \infty)$ and measurable on $t \in [0, \infty)$ for each $z \in \mathbb{D}$ satisfying $\operatorname{Re} p(z, t) > 0$ for all $z \in \mathbb{D}$ and $t \in [0, \infty)$. The theory which centers round Loewner chains and this type of differential equations is called *Loewner theory*, originated with a work by Loewner in 1923, and making remarkable advances in the last decade, including the celebrated *Schramm-Loewner evolution*.

In 1972, Becker [1] proved an interesting relation between Loewner theory and quasiconformal mappings. If a Herglotz function p determined by a Loewner chain f_t as above satisfies

$$\left| \frac{1 - p(z, t)}{1 + p(z, t)} \right| \leq k$$

for all $z \in \mathbb{D}$ and almost all $t \in [0, \infty)$, then f_0 has a k-quasiconformal extension to the complex plane $\mathbb{C}$, that is, there exists a k-quasiconformal mapping $F : \mathbb{C} \to \mathbb{C}$ such that $F\big|_{\mathbb{D}} \equiv f_0$.

In this talk, we discuss the Becker's result will be generalized for L^d-Loewner chains, a general version of the notion of Loewner chains introduced in [3] in a deep connection with L^d-evolution families, developed in [2].

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On (q, p)-Poincaré inequalities for quasihyperbolic boundary condition domains

We study the validity of (q, p)-Poincaré inequalities, when $1 \leq q < p$, in irregular domains with a logarithmic growth condition for the quasihyperbolic metric. These domains are examples of s-John domains, $s > 1$. In particular, we show that for each domain there exists an explicit number p_0 which depends on q, on the logarithmic growth condition, and on the boundary of the domain, such that the (q, p)-Poincaré inequality holds whenever $p > p_0$.

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■ **Valentina Iakovleva** Universidad Simón Bolívar, Departamento de Matemáticas, Caracas - Venezuela, email: romanova@usb.ve, **Judith Vanegas** Universidad Simón Bolívar, Departamento de Matemáticas, Caracas - Venezuela, email: cvanegas@usb.ve

On the Solution of a Mixed-Type Differential Difference Equation

In this talk we will show the construction of the solution to the mixed type differential difference equation:

$$x'(t) = ax(t + \tau) + bx(t - \tau) + cx(t), \quad (1)$$

where $a, b, c \in \mathbb{C} \setminus \{0\}$, $\tau > 0$.

We use a method of derivation steps so that the following condition

$$\varphi^{(n+1)}(0) = a\varphi^{(n)}(\tau) + b\varphi^{(n)}(-\tau) + c\varphi^{(n)}(0)$$

on the the initial function $\varphi \in C^{\infty}[-\tau, \tau]$ assures the existence and uniqueness of the smooth solution of (1).

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Boundary value problems for holomorphic functions in spaces with modulus of continuity

We consider Riemann boundary value problem

$$\Psi^{+}(t) = G(t)\Psi^{-}(t) + g(t), t \in L,$$

where contour $L = \{z \in \mathbb{C} : |z| = 1\}$ is dividing extended complex plane into interior set D^+ and exterior set D^-, g, $G : L \to \mathbb{C}$ are given continuous functions, $G(t) \neq 0$ for any $t \in L$, $\Psi^+ \in H(D^+) \bigcap C(D^+ \cup L)$, $\Psi^- \in H(D^-) \bigcap C(D^- \cup L)$ are unknown functions. It is known ([1], [2]) that in the case of Hölder continuous on L functions g, G there is a way to obtain solution of the problem. Using the same approach we prove that the problem can be solved with more general assumptions regarding functions g and G, namely when the functions belong to space with modulus of continuity with special properties. Moreover, we show that behavior of solution can also be expressed in terms of spaces with modulus of continuity and such called "logarithmic effect" ([3]) can appear. Further, we consider and solve Riemann–Hilbert and Dirichlet boundary value problems by reducing to the researched Riemann boundary value problem.

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Boundedness of pseudodifferential operators on Lebesgue spaces with mixed norm

The continuity of pseudo-differential operators, with symbols in Hörmander's class $S^0_{\rho,\delta}$, on the Lebesgue spaces was studied by numerous authors decades ago, and complete results can be found in a number of standard monographs today. These results can be extended to pseudo-differential operators of non-zero order (symbols in $S^m_{\rho,\delta}$) on Sobolev spaces, allowing for numerous applications in the theory of partial differential equations.

However, in some problems arrising in partial differential equations, like fine studies of elliptic regularity, or for evolution equations, results involving Sobolev spaces with mixed-norm would be of significant interest. These spaces are based on the Lebesgue spaces with mixed-norm, that were introduced by Benedek and Panzone (1961), while some further properties (including a version of the Marcinkiewicz interpolation theorem) were obtained by Russian school (S. M. Nikol'skiĭ and collaborators) a few years later.

We shall prove the boundedness result for pseudo-differential operators with symbols in $S^0_{1,\delta}$, $0 \leq \delta < 1$, on the Lebesgue spaces with mixed norm, extending the classical proof based on Calderón-Zygmund decomposition. Furthermore, some applications to the partial differential equations will be discussed as well.

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Ill-posedness for the Schrödinger equations in one and two space dimensions

We consider the Cauchy problems for the Schrödinger equations with a quadratic nonlinearity and study the ill-posedness by showing that the continuous dependence on initial data does not hold in general. In one space dimension, the critical regularity in Sobolev space $H^s(\mathbb{R})$ is $s = -1$ for the well-posedness and the ill-posedness, and this was shown by Bejenaru-Tao (2006). We introduce Besov spaces to show the ill-posedness in $B^{-1}_{2,q}(\mathbb{R})$ $(q > 2)$ which are larger than $H^{-1}(\mathbb{R})$. In two space dimensions, the regularity $s = -1$ is critical from the view point of the scaling invariance and the problem is ill-posed in $H^{-1}(\mathbb{R}^2)$.

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Construction of the fundamental solution for a Grushin type operator and its application

We will give a method to obtain a precise expression of the fundamental solution for a degenerate operator of Grushin type. We note that this method can be applied to obtain the fundamental solution of Kohn-Laplacian model which is studied in [1] and [2].

The key point of our method is to express the fundamental solution of a degenerate heat equation in the polar coordinates by the modified Bessel function.

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and **Karen Yagdjian** University of Texas-Pan American, USA, email: yagdjian@utpa.edu

Recent results on uniqueness and continuous dependence in the Cauchy problem for backward-parabolic operators with low-regular coefficients

We consider the Cauchy problem for backward-parabolic operators

$$\mathcal{P}u = \partial_t u + \sum_{i,j=1}^{n} \partial_{x_i}(a_{i,j}(t,x)\partial_{x_j}u) + \sum_{k=1}^{n} b_k(t,x)\partial_k u + c(t,x)u$$

and look for sufficient and necessary conditions to ensure the uniqueness of the solutions or the continuous dependence of the solutions on the Cauchy data. We are especially interested in the connections between the regularity of the principal part coefficients and the mentioned properties. It is almost classical (Lions/Malgrange) that the questions about uniqueness and stability have a positive answer if $a_{i,j}(t,x)$ are Lipschitz continuous with respect to time and bounded with respect to the spatial variable. We follow two possibilities to weaken the Lipschitz condition:

(P1) *Local irregularity*: Suppose that $a_{ij} = a_{ij}(t)$ with $|F(t)d_t a_{ij}(t)| \le C_{small}$, where F is a suitable function like $F(t) = t$ or $F(t) = t^2$.

(P2) *Global irregularity*: Suppose that $a_{ij} = a_{ij}(t,x) \in C^{\mu}([0,T], L^{\infty}(\mathbb{R}^n) \cap L^{\infty}([0,T], C^{\omega}(\mathbb{R}^n))$, where we investigate also the possible interactions between ω and μ.

We will illustrate the necessity of our conditions by suitable counterexamples. To prove our results we will use the Carleman estimate method and to prove suitable Carleman estimates we will use, e.g, Bony's para-differential calculus. the results about local irregularity are connected to singular Carleman weight functions. We will conclude the talk with some recent result obtained with K. Yagdjian on uniqueness in the Cauchy problem for degenerate elliptic operators generalizing a zone-method approach developed by Yagdjian.

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THE APPROXIMATE SOLUTION OF THE TASK ON STRUCTURE OF SHOCK WAVES IN GASES FOR FINAL NUMBERS OF THE MOVE.

UDC 533.6.011

In work the approximate solution of a task on structure of shock waves is provided in gases. Changes of average sizes of density, speed and internal energy of gas along width of a shock wave are given. Dependence of width of jump on Move number which is compared to results of other authors is received.

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The initial values problem for generalized metamonogenic initial functions

Consider the initial value problem of the type

$$\frac{\partial \omega}{\partial t} = L\left(t, x, \omega, \frac{\partial w}{\partial x_i}\right) \quad (1)$$

$$\omega(0, x) = \psi(x) \quad (2)$$

where t is the time, L is a linear first order operator in a Clifford Analysis and ψ is a first order generalized metamonogenic function. In this work we give sufficient conditions on the coefficients of operator L under which L is associated to differential equations with anti-metamonogenic right-hand sides. For such operator L the initial problem (1), (2) is solvable for an arbitrary generalized metamonogenic initial function ψ and the solution is also generalized metamonogenic for each t.

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The Fejer - Riesz type results for some weighted Bergman spaces in the unit disc

In this lecture we describe the analogues of the well-known Fejer - Riesz inequality in the classical Hardy space of analytic functions for some weighted Bergman spaces of analytic functions in the unit disc. We describe also a class of such spaces for which the Fejer - Riesz inequality type results do not hold.

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Neutral coated inclusions of finite conductivity

We discuss conductivity of two-dimensional media with coated neutral inclusions of finite conductivity. Such an inclusion when inserted in a matrix does not disturb the uniform external field. We are looking for shapes of the core and coating in terms of the conformal mapping $\omega(z)$ of the unit disc onto coated inclusions. The considered inverse problem is reduced to an eigenvalue problem for an integral equation containing singular integrals over a closed curve L_1 on the transformed complex plane. The conformal mapping $\omega(z)$ is constructed via eigenfunctions of the integral equation. For each fixed curve L_1 the boundary of the core is given by the curve $\omega(L_1)$. The boundary of the coating is obtained by the mapping of the unit circle. It is justified that any shaped inclusion with a smooth boundary can be made neutral by surrounding it with an appropriate coating. Shapes of the neutral inclusions are obtained in analytical form when L_1 is an ellipse.

The results obtained allows us to make the following conclusion. Any two-dimensional core, i.e., a core of an arbitrary smooth shape and of an arbitrary conductivity can be coated by such a material that the coated inclusion inserted in a matrix of an arbitrary fixed conductivity does not disturb the uniform field outside the inclusion.

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Boundary Properties and Biorthogonal Systems in Spaces $A^2_\omega \subset H^2$

The talk is devoted to an investigation of some Hilbert spaces A^2_ω which are contained in Hardy's H^2 of functions holomorphic in $|z| < 1$. In particular, it is proved a theorem stating that the boundary properties of functions from A^2_ω are characterized by some ω-capacities and some biorthogonal systems of functions containing small than one degrees of the Cauchy kernel are found in A^2_ω by means of an explicit isometry between A^2_ω and H^2.

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On Banach Spaces of Functions Delta-Subharmonic in the Unit Disc

The talk is describing some investigations finally aimed at partitioning the whole class of functions delta-subharmonic in the unit disc into some Banach spaces.

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On convergence of double Fourier series with monotone coefficients

Dyachenko ([1]) investigated convergence of double trigonometric Fourier series

$$\sum_{m_1,m_2=1}^{\infty} a_{m_1,m_2} e^{im_1 x} e^{im_2 y}, \tag{1}$$

of a function $f(x,y)$ in the space $L_p(0,2\pi)^2$, $1<p<\infty$.

Theorem A. Let $p>2$. If a function $f(x,y)\in L_p(0,2\pi)^2$ has trigonometric Fourier series (1) with monotone coefficients on each index, then this series convergence in the Pringsheim sense everywhere on $(0,2\pi)^2$.

Let $\mathbf{p}=(p_1,p_2)$, where $1<p_1,p_2<\infty$. The space $L_{\mathbf{p}}(0,1)^2$ is defined as follows

$$\|f\|_{L_{\mathbf{p}}} = \left(\int_0^1 \left(\int_0^1 |f(x,y)|^{p_1} dx\right)^{p_2/p_1} dy\right)^{1/p_2} < \infty.$$

Theorem 1. *Let* $\mathbf{p}=(p_1,p_2)$*, where* $1<p_1,p_2<\infty$ *and* $\frac{1}{p_1}+\frac{1}{p_2}<1$*. If the series*

$$\sum_{m_1,m_2=0}^{\infty} a_{m_1,m_2}\varphi_{m_1}(x)\varphi_{m_2}(y),$$

where $\varphi_m(x)\in\Phi=\{\varphi_m(x)\}_{m=1}^{\infty}$ *is a regular system (see* [2]*), is the Fourier series of a function* $f\in L_{\mathbf{p}}(0,1)^2$ *such that coefficients are monotone on each index, then the series converges in the Pringsheim sense everywhere on* $(0,1)^2$.

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WEAK SOLUTIONS FOR THE SINGULAR POTENTIAL WAVE SYSTEM

We prove that the following class of the system of the nonlinear wave equations with singular potential nonlinear term

$$U_{tt}-U_{xx}=G_U(x,t,U) \qquad \text{in } \left(-\frac{\pi}{2},\frac{\pi}{2}\right)\times R,$$

$$u_i\left(\pm\frac{\pi}{2},t\right)=0,\quad u_i(x,t)=u_i(-x,t)=u_i(x,-t)=u_i(x,t+\pi),\quad i=1,\dots,n,$$

where $U=(u_1,\dots,u_n)$ and $G\in C^2([-\frac{\pi}{2},\frac{\pi}{2}]\times R^1\times D, R^1)$ satisfies the following conditions:

$(G1)$ There exists $R_0>0$ such that

$$\sup\{|G(x,t,U)|+\|\mathrm{grad}_U G(x,t,U)\|_{R^n}|\ (x,t,U)\in[-\frac{\pi}{2},\frac{\pi}{2}]\times R^1\times(R^n\backslash B_{R_0})\}<+\infty.$$

$(G2)$ There is a neighborhood Z of C in R^n such that

$$G(x,t,U)\geq\frac{A}{d^2(U,C)} \quad \text{for} \quad (x,t,U)\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\times R\times Z,$$

where D is an open subset in R^n with compact complement $C = R^n \backslash D$, $n \geq 2$, has at least one nontrivial weak solution.

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On some structural sets and a quaternionic (φ, ψ)-hyperholomorphic function theory

Quaternionic analysis is regarded as a broadly accepted branch of classical analysis referring to many different types of extensions of the Cauchy-Riemann equations to the quaternion skew field $\mathbb{H}$. It relies heavily on results on functions defined on domains in $\mathbb{R}^4$ (or $\mathbb{R}^3$) with values in $\mathbb{H}$. One of this extensions is the concept of $\psi-$hyperholomorphic functions related to a so-called structural set ψ of $\mathbb{H}^4$ (or $\mathbb{H}^3$) respectively. This concept was introduced by M. Shapiro and N. Vasilevski [3]. Later on in the study of a generalized Π-operator K. Gürlebeck, U. Kähler, and M. Shapiro required two structural sets, which includes and uniformizes several classic cases, such as D and $\overline{D}$. The main goal of this talk is to present the nucleus of a (φ, ψ)-hyperholomorphic function theory, i.e., simultaneous null solutions of two Cauchy-Riemann operators associated to a pair φ, ψ of structural sets of $\mathbb{H}^4$. Following a matrix approach, a generalized Borel-Pompeiu formula and the corresponding Plemelj-Sokhotzki formulae are established.

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Boundary conditions of classical potentials and its applications

Problems of finding boundary conditions of classical volume potentials such as Newton, heat and wave potentials are stated. The problems has been solved i.e., find corresponding non–local boundary value problem which is correct and its unique solution coincides with volume potentials. And its applications were investigated.

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On various classes of approximate units and the convolution of generalized functions

General sequential conditions of integrability and convolvability in various spaces of generalized functions, expressed in terms of different classes of approximate units, are discussed. The equivalence of the respective convolutions of generalized functions is proved.

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On the existence and growth of the convolution of functions and generalized functions

We extend and essentially sharpen our earlier results concerning the existence and growth of the convolution in various spaces of functions and generalized functions. The applications of the results in the theory of integral transforms are shown.

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Lyapunov exponents for bundles over ball quotients and commensurability among lattices

A ball quotient is the orbit space of the action of a lattice in $\mathrm{PU}(1,n)$ on complex hyperbolic n-space. To any flat vector bundle V over a ball quotient equipped with a suitable norm, we can associate a set of real numbers, the Lyapunov exponents. These describe the logarithmic growth of sections when dragged along a generic geodesic. There is a deep link between dynamics and algebraic geometry that relates the sum of the Lyapunov exponents to ratios of Chern classes of certain vector bundles, which was first observed by Kontsevich and Zorich for the Teichmüller geodesic flow on the moduli space of curves. We prove a variant of the Kontsevich-Zorich formula for ball quotients in the case that the flat bundle comes from algebraic geometry, i.e. carries a variation of Hodge structures.

For an application of our formula, we consider the non-arithmetic lattices found by Deligne and Mostow. They come from families of cyclic coverings of the projective line. Taking the lattice and its Galois conjugates gives rise to a flat vector bundle which is a direct factor in the variation of Hodge structures of this family of curves. Using our formula and the explicit description of the period map, we compute all the individual Lyapunov exponents of this bundle.

On the other hand, we show that Lyapunov exponents are commensurability invariants of the lattice. Since the Deligne-Mostow lattices account for all presently known commensurability classes of non-arithmetic lattices, our results complete the commensurability classification begun by Deligne and Mostow.

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Recent topics in fixed point theory

In this talk, we discuss the recent trends in metric fixed point theory which was initialized by Banach [3]. In particular, we consider the correspondence between the partial metric space [4]-[10] and usual metric space and further G-metric space[2]-[13] and usual metric space. We examine the connection between metric like space [1] (dislocated metric space [5, 6]) and partial metric space. We also mention the consequences of these relation to the fixed point results.

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On α-embedded sets

Let $\mathcal{G}_0^*(X)$ and $\mathcal{F}_0^*(X)$ be the collections of all functionally open and functionally closed subsets of a topological space X, respectively. Assume that the classes $\mathcal{G}_\xi^*(X)$ and $\mathcal{F}_\xi^*(X)$ are defined for all $\xi < \alpha$, where $0 < \alpha < \omega_1$. Then, if α is odd, the class $\mathcal{G}_\alpha^*(X)$ ($\mathcal{F}_\alpha^*(X)$) consists of all countable intersections (unions) of sets of lower classes, and, if α is even, the class $\mathcal{G}_\alpha^*(X)$ ($\mathcal{F}_\alpha^*(X)$) consists of all countable unions (intersections) of sets of lower classes. The classes $\mathcal{F}_\alpha^*(X)$ for odd α and $\mathcal{G}_\alpha^*(X)$ for even α are said to be *functionally additive*, and the classes $\mathcal{F}_\alpha^*(X)$ for even α and $\mathcal{G}_\alpha^*(X)$ for odd α are called *functionally multiplicative*.

A subset E of a space X is *α-embedded* in X if for any set A of the α'th functionally additive class in E there is a set B of the α'th functionally additive class in X such that $A = B \cap E$.

Theorem 1. *Let X be a topological space, $E \subseteq X$ and*

(1) *X is perfectly normal, or*
(2) *X is completely regular and E is its Lindelöf subset, or*

(3) *E is a functionally open subset of X, or*

(4) *X is a normal space and E is its F_σ-subset,*

then E is α-embedded in X for each $0 \le \alpha < \omega_1$.

Theorem 2. *There exist a completely regular space X and its 1-embedded subspace $E \subseteq X$ which is not 0-embedded in X.*

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Pseudo-differential and convolution type operators with non-regular symbols

The talk is devoted to studying pseudo-differential operators with non-regular symbols on weighted Lebesgue spaces. These symbols consist of all bounded measurable or piecewise continuous $V(\mathbb{R})$-valued functions on $\mathbb{R}$, or of all Lipschitz $V_d(\mathbb{R})$-valued functions on $\mathbb{R}$, where $V(\mathbb{R})$ is the Banach algebra of all functions of bounded total variation on $\mathbb{R}$, and $V_d(\mathbb{R})$ is the Banach algebra of all functions of bounded variation on dyadic shells on $\mathbb{R}$. Thus, such symbols admit discontinuities with respect to spatial and dual variables, which leads to non-commutative Fredholm symbol calculi for corresponding pseudo-differential operators.

Applying the theory of pseudo-differential operators with non-regular symbols, we study the Banach algebras $\mathfrak{A}_{p,w}$ generated by all convolution type operators of the form $aF^{-1}bF$ on weighted Lebesgue spaces $L^p(\mathbb{R}, w)$ where $1 < p < \infty$ and w are Muckenhoupt weights, F is the Fourier transform, the functions $a, b \in L^\infty(\mathbb{R})$ admit piecewise slowly oscillating discontinuities on $\mathbb{R} \cup \{\infty\}$, and b is a Fourier multiplier on $L^p(\mathbb{R}, w)$. A Fredholm symbol calculus is constructed and a Fredholm criterion for the operators $A \in \mathfrak{A}_{p,w}$ in terms of their Fredholm symbols is established. This part of the talk is based on a joint work with I. Loreto-Hernández.

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Toeplitz and Wiener-Hopf operators with oscillating matrix symbols

The talk is devoted to the Fredholm theory of Toeplitz and Wiener-Hopf operators with oscillating matrix symbols generated by semi-almost periodic and slowly oscillating matrix functions on the scale of weighted Lebesgue spaces. The study is based on the theory of pseudodifferential and Calderón-Zygmund operators, the Allan-Douglas local method, the limit operators method and different approaches to almost periodic factorization of almost periodic matrix functions. In particular, for new classes of triangular almost periodic matrix functions, we study the existence of such a factorization and construct it explicitly on the basis of almost periodic solutions of corresponding Riemann-Hilbert problems, applications of the corona problem and the ergodic theory. This allows one to elaborate also the solvability theory for Toeplitz and Wiener-Hopf operators with corresponding almost periodic matrix symbols.

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Boundary Control of a Neutral Type Hyperbolic Equation

We study boundary control problems for a system described by a hyperbolic type equation with time delays in the higher–order terms. In the theory of differential–difference equations such equations are called neutral type equations. For fixed $\tau > 0$ and $b \in \mathbb{R}$ we consider the initial boundary value problem described by the equation

(1) $y_{tt}(x,t) - (p(x)y_x(x,t))_x + b\,[y_{tt}(x,t-\tau) - (p(x)y_x(x,t-\tau))_x] = 0;\ 0 < x < X,\ 0 < t < T,$

with the boundary conditions

$$y(0,t) = 0,\ y_x(X,t) = u(t);\ 0 < t < T,\ u \in L^2(0,T), \quad (2)$$

and the initial conditions

$$\begin{cases} y(x,0) = y_0(x),\ y_t(x,0) = y_1(x);\ 0 < x < X, \\ y_{tt}(x,t-\tau) - (p(x)y_x(x,t-\tau))_x = g(x,t);\ 0 < x < X,\ 0 < t < \tau. \end{cases} \quad (3)$$

Here $p \in C^1[0,X]$, $p(x) > 0$, $y_0 \in \mathcal{H}^1 := \{f \in H^1(0,X) : f(0) = 0\}$, $y_1 \in L^2(0,X)$, $g \in L^2((0,X) \times (0,\tau))$.

We prove that the initial boundary value problem (1) – (3) has the unique generalized solution such that $y \in C([0,T];\mathcal{H}^1)$, $y_t \in C([0,T];L^2(0,X))$. We demonstrate also the exact controllability of the system for $T \geq T_0 := 2\int_0^X [p(x)]^{-1/2}dx$. More precisely, we prove that for any given y_0, y_1, g from the corresponding spaces, there exists a boundary control $u \in L^2(0,T)$ such that $y(\cdot,T) = y_t(\cdot,T) = 0$, and this control is unique if $T = T_0$.

Then, for any $T > 0$, we consider minimization problem for the functional

$$J(u) = \int_0^X [y_t^2(x,T;u) + p(x)y_x^2(x,T;u)]\,dx + \gamma\int_0^T u^2(t)\,dt \quad (4)$$

with arbitrary fixed $\gamma > 0$. We prove the existence and uniqueness of the solution u_* of this problem and propose the algorithm of its construction. Our approach is based on reducing the control problem to a moment problem and investigation of Riesz bases of the corresponding *quasi-exponential* families.

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Well–posedness of coupled first and second order hyperbolic systems

Existence, uniqueness and well-posedness of quasilinear wave equations that are coupled to first order symmetric hyperbolic systems are established. Hughes, Kato and Marsden studied systems consisting only of second order hyperbolic equations [1]. They proved existence theorems for systems in $\mathbb{R}^n$ whose solutions $(u(t), \partial_t u(t))$ lie in the Sobolev space $H^{s+1} \times H^s$. In particular, they lowered the required value of s to $s > \frac{n}{2}$ in case where the coefficients of highest order terms do not involved derivatives of the unknowns. We extent their results and obtain the same degree of regularity for coupled first and second oder hypebolic systems. This result is applied to show short time existence theorems for Cauchy problems for non–vacuum Einstein equations. This is a joint work with Uwe Brauer.

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On boundary properties of conformal homeomorphisms

Let suppose that a simply connected domain in the complex plane bounded by a smooth Jordan curve is given. Let consider a homeomorphism of the closed unit disk onto the closure of this domain conformal in the open unit disk. Let consider the angle between the tangent to the curve and the positive real axis as the function of the arc length on the curve.

Kellog in 1912 proved that if this angle satisfies Hölder condition, then the derivative of the function realizing conformal mapping satisfies Hölder condition with the same index. Connection between properties of the boundary of the domain and properties of the considered function was investigated in works by several authors. In particular, some results were received by author in terms of moduli of smoothness of different types.

We represent some new estimates for derivatives of the functions realizing conformal mapping on the boundary of the domain formulated in terms of integral moduli of smoothness introduced by P. M. Tamrazov in 1977. Difference between these moduli and traditional integral moduli of smoothness, introduced as the least upper bound of averaging absolute values of finite differences is that the operators of averaging and taking of least upper bound are applied in reverse order.

Let two simply connected domains bounded by the smooth Jordan curves be given. Let characterize boundaries of these domains by the angles between the tangent to the curves and the positive real axis considered as the functions of the arc length on the curves.

We represent new estimates for integral moduli of smoothness of arbitrary order for derivatives of homeomorphisms between the closures of the considered domains conformal in open domains. In partial case when one of considered domains is the closed unit disk we receive generalizations and inversations of Kellogs type theorems.

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Self-Similarity vs Smoothness

We consider a closed simple curve Γ and a function $f(t)$ satisfying the Hölder condition with exponent $\nu \in (0, 1]$ on this curve. If the curve Γ is rectifiable, then we denote by $\Phi(z)$ the Cauchy integral with density f over Γ. If Γ is non-rectifiable, then Φ stands for the Cauchy transform of distribution $f\overline{\partial}\chi$, where χ is characteristic function of interior of Γ (see [1]).

If Γ is rectifiable, then the Cauchy integral $\Phi(z)$ has continuous boundary values on Γ from the both sides for $\nu > \frac{1}{2}$ (see [2, 3]). If, additionally, the curve is piecewise smooth, then the same is valid for $\nu > 0$ (see, for instance, [4]).

If Γ is not rectifiable, then the Cauchy transform $\Phi(z)$ has continuous boundary values for $\nu > \frac{d}{2}$, where d is upper metric dimension [5] of the curve Γ (see [1]). In the present report we prove that for self-similar non-rectifiable curves the same is valid for $\nu > d - 1$. In other words, the self-similarity improves the boundary properties of $\Phi(z)$ in just the same degree as the smoothness for rectifiable one.

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The Marcinkiewicz exponent with applications

We introduce the following new metric characteristic of non-rectifiable curves on the complex plane. Let Γ be a closed Jordan curve of null area dividing complex plane into finite domain D^+ and infinite domain D^-. We put

$$\mathfrak{me}(\Gamma) := \sup\left\{p : \iint\limits_{D^+} \frac{dxdy}{dist^p(z,\Gamma)} < \infty\right\}$$

and

$$\mathfrak{me}^-(\Gamma) := \sup\left\{p : \iint\limits_{D^*} \frac{dxdy}{dist^p(z,\Gamma)} < \infty\right\}.$$

Here D^* is domain $D^- \bigcap \{z : |z| < r\}$, where r is so large that Γ completely lies in the disk $\{z : |z| < r\}$. We call these values the Marcinkiewicz exponents of the curve Γ in connection with Marcinkiewicz's researches of plane and spatial sets in terms of certain integrals over the complements of these sets (see, for instance, [1]).

We apply these characteristics for solving of the Riemann boundary value on non-rectifiable curves. Let us cite a result concerning the jump problem, i.e., the problem on evaluation of holomorphic in $\overline{\mathbb{C}} \setminus \Gamma$ function Φ satisfying boundary condition

$$\Phi^+(t) - \Phi^-(t) = g(t), \quad t \in \Gamma,$$

where function g (the jump) is given.

Theorem 1. *If the jump g satisfies on Γ the Hölder condition with exponent ν and*

$$\nu > 1 - \frac{1}{2}\max\{\mathfrak{me}(\Gamma), \mathfrak{me}^{-}(\Gamma)\},$$

then the jump problem is resolvable.

We construct examples showing that this solvability condition for the jump problem on non-rectifiable curve is sharper than earlier known ones (see [2, 3]).

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ON THE WELL-POSSED OF GENERAL NONLINEAR BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF IMPULSIVE EQUATIONS WITH FINITE AND FIXED POINTS OF IMPULSES

The general nonlocal boundary value problem is considered for systems of impulsive equations with finite and fixed points of impulses actions. Sufficient conditions are given for the solvability and unique solvability of this problem.

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Chow stability and the projectivisation of stable bundles

It appeared in the work of S.K. Donaldson that the notion of asymptotic Chow stability is central in the problem of existence of constant scalar curvature Kähler metric on a polarized manifold. In this talk, we are interested in ruled manifolds, i.e projectivisation of a vector bundle over a polarized manifold. In the recent years, ruled manifolds have been a great source of understanding of the so-called Yau-Tian-Donaldson conjecture relating stability and existence of metrics with special curvature properties. We will discuss the Chow stability of the projectivisation of a Gieseker stable bundle over a surface endowed with a constant scalar curvature Kähler metric. In particular, we will provide an example of a smooth manifold which is Chow stable but not asymptotically Chow stable. We will also provide examples of ruled manifolds that are K-(poly)stable with respect to certain polarisations but are K-unstable with respect to other polarisations. This is a joint work with J. Ross (Cambridge University).

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Queue-size distribution in energy-saving model based on multiple vacation policy

The problem of energy saving is extremely important in wireless sensor networks or mobile nodes (stations) of WiMAX. Indeed, majority of sensor nodes are equipped with non-rechargable batteries. To prolong the lifetime of the battery different wake-up strategies are being proposed in the literature, focused on the optimal increasing the probability of sleeping state (see e.g. [2], where the model with N-policy is discussed). Similarly, in the IEEE 802.16e standard of mobile WiMAX three different classes of power-saving mechanisms are defined (see e.g. [1], [8]). Since, during the power-saving (sleep) mode, the power consumed is much smaller than in the wake (listening) mode, the key problem is the efficient reduction the wake mode duration. An $M/G/1$ queue with infinite buffer, multiple vacation policy and exhaustive service is presented in [7] as a model of Type I power-saving mode, and some performance measures are derived there.

In the paper we investigate the transient queue-size distribution in the model of energy-saving mechanism based on the $M/G/1/N$-type finite-buffer queue with repeated (multiple) independent vacations. Every time when the server becomes idle it takes successive vacations until, at the end of one of them, at least one packet waits for service. Then, after finishing this vacation, a busy period begins immediately, during which the queue empties. A similar model with infinite buffer was studied in [3] and [4], where the formulae for transforms of the queue-size distribution and departure process were obtained, respectively.

For the transient queue-size distributions, conditioned by the numbers of packets present in the system at the opening, we build a system of integral equations, using the paradigm of embedded Markov chain. Using the approach proposed in [6], we obtain the general solution of the corresponding system written for Laplace transforms in a compact form. Numerical examples are attached as well.

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About the solvability of a nonlinear optimization of the processes described by integro-differential equations

The optimal control problem of distributed parameter systems described by integro-differential equations has been analyzed. We consider controlled process describable by scalar function $V(t,x)$, which in the domain $Q_T = Q \times (0,T)$ satisfies to equation in

$$V_t = AV + \int_0^T K(t,\tau)V(\tau,x)d\tau + g(t,x)f[t,u(t)], \quad (1)$$

to the initial and boundary conditions

$$V(0,x) = \psi(x), x \in Q, \quad (2)$$

$$\Gamma V(t,x) \equiv \sum_{i,j=1}^{n} a_{ij}(x)V_{x_j}(t,x)cos(\nu,x_i) + a(x)V(t,x) = 0. \quad (3)$$

Here $f[t,u(t)] \in H(0,T)$ is a given function of external source nonlinearly depending on the control function $u(t) \in H(0,T)$; $Q \subset R^n$ with piecewise smooth boundary $\gamma; \nu$ is outer exterior normal at the point $x \in \gamma$, $g(t,x) \in H(Q_T), \psi(x) \in H(Q)$ are given functions; A is eliptic operator;$a(x) \geq 0; c(x) \geq 0$ are bounded measurable functions; H is Hilbert space.

It is required to find an admissible pair $(u^0(t), V^0(t,x) \in H(0,T) \times H(Q_T))$ on which given functional $I[u(t)]$ is minimized.

We have found the conditions of control optimality and sufficient conditions of a unique optimal control existence, which is defined as the solution of a nonlinear integral equation and satisfies to the additional condition in the form of inequality. The algorithm for constructing of nonlinear optimization task solution and its approximations has been worked out. We have proved the convergence of the approximate solutions.

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Rearrangement Invariant Sobolev Spaces on General Domains

This talk concerns Sobolev spaces of differentiable function on finite measure domains in $\mathbb{R}^n$. Such a space is determined by a rearrangement invariant (r.i.) functional, ρ, like those of Lebesgue, Lorentz and Orlicz. We have two goals. First, we seek a functional to describe the smallest set that contains the decreasing rearrangements of functions in an r.i. Sobolev space. Second, we study refinements of Sobolev-Poincaré imbedding inequalities which, for spaces of functions with first order derivatives, have the form

$$\inf_{c \in \mathbb{R}} \sigma(u-c) \leq A\rho(|\triangle u|).$$

Here, σ is another r.i. functional, which we would like to be as large as possible. It turns out that for the spaces of functions with higher order derivatives the two problems are connected.

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Linear Differential Equations in the Algebra of Almost Periodic Generalized Functions

This work introduces and studies an algebra of almost periodic generalized functions generalizing trigonometric polynomials, classical almost periodic functions as well as almost periodic

Schwartz distributions. Then we study a linear system of ordinary differential equations in this algebra of almost periodic generalized functions.

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Complex spaces and nonstandard schemes

The value of nonstandard mathematics [4] is in serving as a "guiding star" and often offering a conceptually simple and elegant interpretation and generalization of classical theory and sometimes leads to new concrete standard results.
Not so much is known about nonstandard complex analysis, unlike nonstandard real analysis [1], topology and metric spaces theory[3]. Only some very specific applications of model theory are used to be known as for instance the Lefschetz principle, the theorem of Tarski-Seidenberg or some simple proofs of Hilbert's Nullstellensatz.

Recently, in collaboration with S. Kosarew, we started a program to develop a theory of analytic geometry using nonstandard methods. One of our fundamental constructions is that of a category of certain ringed spaces, called bounded schemes, which contains the category of algebraic $\mathbb{C}$- schemes and which admits an essentially surjective functor, called the standard part functor, to the category of complex spaces. The advantage of this new more algebraic category is that it allows us to apply many constructions of standard algebraic geometry which are not evident in the analytic context. We obtain analytic results just by taking the standard part functor. The following tasks have been successfully carried out in our paper [2]:

- Holomorphic functions should be the standard part of polynomials of hyperfinite degree
- Complex spaces should be seen as hyperalgebraic schemes
- Generic points of irreducible complex spaces/schemes should be certain nonstandard points
-Differential forms as functions taking infinitesimal values (Leibniz vision)

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A constructive method for an approximate solution to scalar Wiener–Hopf equations

This paper presents a novel method of approximating the scalar Wiener–Hopf equation, and therefore constructing an approximate solution. The advantages of this method over the existing methods are reliability and explicit error bounds. Additionally, the degrees of the polynomials in the rational approximation are considerably smaller than in other approaches. The need for a numerical solution is motivated by difficulties in computation of the exact solution. The approximation developed in this paper is with a view of generalization to matrix Wiener–Hopf problems for which the exact solution, in general, is not known. The first part of the paper develops error bounds in L_p for $1 < p < \infty$. These indicate how accurately the solution is approximated in terms of how accurately the equation is approximated. The second part of the paper describes the approach of approximately solving the Wiener–Hopf equation that employs the rational Carathéodory–Fejér approximation. The method is adapted by constructing a mapping of the real line to the unit interval. Numerical examples to demonstrate the use of the proposed technique are included (performed on Chebfun), yielding errors as small as 10^{12} on the whole real line.

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Differential Operators, Integral Formulae and Symmetries

Many mathematical models of physical problems possess certain group of symmetries. This symmetries are inherited by differential equations representing the model. We discuss a procedure which supplies an integral formula producing solutions to the differential equation out of the respective symmetry group. The model example is the Cauchy integral formula producing null-solutions of the Cauchy–Riemann equations. We produce the rest of formulae connected with the Möbius transformations of the real line.

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Toeplitz Operators and Covariant Transform

Berezin showed that the co- and contravariant calculus of operators helps to study Toeplitz operators. Both main examples considered by Berezin—the Bergman and the Segal–Bargmann–Fock spaces—are related to certain groups. We review some connections between the representation theory of groups and operator calculus. In particular, we revisit Simonenko's localisation technique.

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Thermoplasticity for the Mróz's model in the thermodynamically complete systems

The subject of our research is a thermomechanical model where the mechanical deformation is coupled with a heat equation. Plastic deformation is consistant with the Mróz model. We provide the proof regarding solutions' existence of thermoplastic model. The considered model is thermodynamically consistent.

We consider the system of equations which describes the thermoelastic body Ω (in $\mathbb{R}^3$) under the action of the external force which causes spatial deformation and changes temperature. After some simplification we get the following system of equations

$$\left\{ \begin{array}{rcl} -\operatorname{div}\mathbf{T} & = & \mathbf{f}, \\ \mathbf{T} & = & \mathbf{D}(\varepsilon - \varepsilon^p), \\ \varepsilon^p_t & = & \mathbf{G}(\theta, \mathbf{T}^d), \\ \theta_t - \Delta\theta & = & \mathbf{T}^d : \varepsilon^p_t, \end{array} \right. \tag{1}$$

with initial and boundary conditions.

Function $\mathbf{G}(\theta, \mathbf{T}^d))$ is the inelastic constitutive function. We make a few assumption on this function and of possible form of $\mathbf{G}(\cdot,\cdot)$ is function $\mathbf{G}(\theta, \mathbf{T}^d) = G_1(\theta)\mathbf{T}^d$ which describes the Mróz model.

By finding a solution of this problem we mean to find the displacement $\mathbf{u}$, plastic strain tensor ε^p and temperature θ of the body. To do this we use two level Galerkin approximation.

This model has two difficulties. Displacement of the body and temperature are related by the constitutive function $\mathbf{G}(\cdot,\cdot)$. Second problem is cause by the assumption on the function $\mathbf{G}(\cdot,\cdot)$. The right hand side of the heat equation is only integrable function, hence, energy estimates are not fulfilled.

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Some questions related to Mizel's problem

We consider the space R^2 and let $A \subset R^2$ be a set of this space.

The set A is called *convex* if it contains, with any two distinct points x and y, the (closed) line segment between x and y. The boundary of A is called *a convex curve.*

Mizel's problem: *Every planar closed convex curve such that, no rectangle has exactly three vertices on it is a circle.*

The set A has *the infinitesimal rectangle property* if there is some $\varepsilon > 0$ such that if any rectangle with sidelengths ratio at most ε has its three vertices on A then this rectangle has the fourth vertex on A.

Theorem *Any convex curve of constant width satisfying the infinitesimal rectangular condition is a circle.*

Corollary *Every Jordan curve satisfying the infinitesimal rectangle property is a circle.*

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Multiparameter bifurcation and symmetry breaking of Solutions of Systems of Elliptic Differential Equations

The aim of my talk is to study behaviour of weak solutions of the following system of elliptic differential equations:

$$(1) \qquad \begin{cases} -\Delta u &= \nabla_u F(u,\Lambda) & \text{in } \Omega \\ u &= (u_1,\ldots,u_m) = 0 & \text{on } \partial\Omega, \end{cases}$$

(1) $\Omega \subset \mathbb{R}^N$ is an open, bounded subset,

(2) $F \in C^2(\mathbb{R}^m \times S(m,\mathbb{R}),\mathbb{R})$ is of the form $F(u,\Lambda) = \frac{1}{2}\langle \Lambda u, u\rangle + \eta(u,\Lambda)$, where

  (a) Λ is a real and symmetric $(m \times m)$-matrix (we denote it $\Lambda \in S(m,\mathbb{R})$),

  (b) $\nabla_u \eta(0,\Lambda) = 0$, for any $\Lambda \in S(m,\mathbb{R})$, (c) $\nabla_u^2 \eta(0,\Lambda) = 0$, for any $\Lambda \in S(m,\mathbb{R})$.

(N4) for any $\Lambda \in S(m,\mathbb{R})$ there are $C_\Lambda > 0$ and $1 \le p_\Lambda < (N+2)(N-2)^{-1}$ such that for any $(u,\Lambda) \in \mathbb{R}^m \times S(m,\mathbb{R})$ the following inequality holds true $\mid \nabla_u^2 F(u,\Lambda) \mid \le C_\Lambda \left(1 + \mid u \mid^{p_\Lambda - 1}\right)$.

The matrix $\Lambda \in S(m,\mathbb{R})$ is considered as a parametr.

The main tool we use is the theory of the degree for SO(2)-equivariant gradient maps defined in [1],[2]. We treat the space $\mathbb{R}^m$ as an orthogonal representation of the group $SO(2)$ and assume that the operator F is $SO(2)$-invariant. Hence the Hilbert space $\mathbb{H} = \bigoplus_{i=1}^{m} \mathbb{H}_0^1(\Omega)$ is an orthogonal representation of the group $SO(2)$ with $SO(2)$-action given by the formula $(g \cdot v)(x) = gv(x)$ for any $g \in SO(2)$ and $v \in \mathbb{H}$.

With the system (1) we associate a C^2-functional $\Phi : \mathbb{H} \times S(m, \mathbb{R}) \to \mathbb{R}$ such that the critical points of Φ are in one-to-one correspondence with a weak solutions of the system. In above situation the operator $\nabla_u \Phi : \mathbb{H} \times S(m, \mathbb{R}) \to \mathbb{H}$ is $SO(2)$-equivariant and self-adjoint; therefore we are able to derive formula for bifurcation index in terms of the degree for $SO(2)$-equivariant gradient operators. Finally, using the Rabinowitz type global bifurcation theory for $SO(2)$-equivariant gradient maps, see [1], we put the sufficient conditions for the existence of global bifurcation points of solutions of (1).

Moreover, we describe a symmetry breaking phenomenon that occurs on continua of nontrivial solutions of (1).

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Robustness of Exponential Dichotomies of Boundary-Value Problems for General First-Order Hyperbolic Systems

We examine robustness of the exponential dichotomy of boundary value problems for linear first-order non-strictly hyperbolic systems of a single spatial variable of the type

$$\partial_t u_j + a_j(x,t)\partial_x u_j + \sum_{k=1}^{n} b_{jk}(x,t)u_k = 0, \quad j \le n,\ x \in (0,1).$$

We show that the dichotomy survives in the space of continuous functions under small perturbations of the coefficients a_j and b_{jk} ([1]). The main technical difficulty lies in showing the smooth dependence of solutions to initial-boundary value problems on the coefficients a_j. To overcome this difficulty, the boundary conditions are supposed to be of types ensuring a smoothing property of solutions [2, 3].

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L^2 boundedness of solutions to the 2D exterior problems for the semilinear heat and dissipative wave equations

We consider the initial boundary value problems for the semilinear heat equations and semilinear dissipative wave equations in two dimensional exterior domains. Also, we consider the Cauchy problems for the linear wave equations with strong damping terms and Navier-Stokes equations in $\mathbf{R}^2$. We will give the L^2 boundedness of the solutions for the initial data in Hardy space. The results in this talk were obtained in a joint work with M. Misawa (Kumamoto University, Japan).

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Extremal properties associated with the generalized Loewner differential equation in $\mathbb{C}^n$

In this talk we survey recent results related to extreme points, support points and reachable families of holomorphic mappings generated by the generalized Loewner differential equation on the unit ball B^n in $\mathbb{C}^n$. Certain applications and conjectures are also considered.

For a linear operator $A \in L(\mathbb{C}^n)$, let $k_+(A)$ be the upper exponential index of A and let $m(A) = \min\{\Re\langle A(z), z\rangle : \|z\| = 1\}$. Under the assumption $k_+(A) < 2m(A)$, we consider the family $S^0_A(B^n)$ of mappings which have A-parametric representation, i.e. $f \in S^0_A(B^n)$ iff there exists an A-normalized univalent subordination chain $f(z,t)$ such that $f = f(\cdot, 0)$ and $\{e^{-tA}f(\cdot,t)\}_{t\geq 0}$ is a normal family on B^n. We are concerned with extreme points and support points associated with the compact family $S^0_A(B^n)$. These results generalize to higher dimensions related results due to Pell and Kirwan. We also present an n-dimensional version of an extremal principle due to Kirwan and Schober, and give applications related to distortion and coefficient bounds for $S^0_{I_n}(B^n)$.

In the second part of the talk, we use ideas from control theory to consider extremal problems related to bounded mappings in $S^0_A(B^n)$. For this aim, we investigate the (normalized) time-$\log M$-reachable family $\tilde{\mathcal{R}}_{\log M}(\mathrm{id}_{B^n}, \mathcal{N}_A)$ generated by the Carathéodory mappings, where $M \geq 1$ and $k_+(A) < 2m(A)$. Every mapping f in this reachable family can be imbedded as the first element of an A-normalized univalent subordination chain $f(z,t)$ such that $\{e^{-tA}f(\cdot,t)\}_{t\geq 0}$ is a normal family and $f(\cdot, \log M) = e^{A\log M}\mathrm{id}_{B^n}$. We present a density result related to the family $\tilde{\mathcal{R}}_{\log M}(\mathrm{id}_{B^n}, \mathcal{N}_A)$, which involves the subset $\mathrm{ex}\,\mathcal{N}_A$ of $\mathcal{N}_A$ consisting of extreme points. These results are generalizations to $\mathbb{C}^n$ of well known results due to Loewner, Pommerenke and Roth. We are also concerned with extreme points and support points associated with compact families generated by extension operators.

This talk is based on joint work with Ian Graham (Toronto), Hidetaka Hamada (Fukuoka) and Mirela Kohr (Cluj-Napoca).

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Poisson problems for semilinear Brinkman systems in Lipschitz domains

The purpose of this talk is to combine a layer potential analysis and the Schauder fixed point theorem in order to show the existence of solutions of a Poisson problem for a semilinear Brinkman system on a bounded Lipschitz domain $\mathfrak{D} \subseteq \mathbb{R}^n$ $(n \geq 2)$ with Dirichlet or Robin boundary condition and data in Sobolev and Besov spaces. The semilinear Brinkman system is written in terms of an essentially bounded Carathéodory function $\mathcal{P}$ from $\mathfrak{D} \times \mathbb{R}^n \times \mathbb{R}$ to $\mathbb{R}^n \otimes \mathbb{R}^n$, which satisfies a nonnegativity condition and a boundedness condition. First, we show the well-posedness of the corresponding linear Poisson problem. Thus, we show the existence and uniqueness of the solution in the aforementioned spaces, together with some useful regularity estimates. Then, by using the well-posedness result from the linear case and the Schauder fixed point theorem, we show the desired existence result for the semilinear Poisson problem. Various applications and particular cases are also presented. In addition, we provide an existence and uniqueness result for the Dirichlet problem associated with the Darcy-Forchheimer-Brinkman system in a bounded Lipschitz domain in $\mathbb{R}^n$ $(n \leq 4)$ with small boundary data.

Note that the semilinear Darcy-Forchheimer-Brinkman equation is a generalization of the Darcy and Brinkman equations for viscous incompressible flows in saturated porous media.

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Approximation of periodic functions in non–standard Banach function spaces

Our talk deals with the approximation problems in some non-standard Banach function spaces. In these spaces we present the fundamental inequalities for the integral functions of finite degree and for trigonometric polynomials. We give various applications to the approximation of periodic functions including a local approximation.

On the base of above–mentioned Banach function spaces we introduce Besov and Liouville type spaces and prove imbedding theorems for these spaces.

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On multiplicative sufficient conditions for Fourier multipliers

New sufficient conditions for Fourier multipliers in the Hardy spaces $H_p(\mathbb{R}^n)$, $p > 0$, and $L_1(\mathbb{R}^n)$ are obtained. These conditions are given in terms of the simultaneous behavior of (quasi-)norms of a function in L_q-space and Besov space $B^s_{r,\infty}$. More specifically, given $m \in L_\infty(\mathbb{R}^n)$ and $\eta \in C^\infty(\mathbb{R}^n)$ such that supp $\eta \subset \{1/4 \le |\xi| \le 4\}$, $0 \le \eta(\xi) \le 1$ for all $\xi \in \mathbb{R}^n$, and $\eta(\xi) = 1$ on $\{1/2 \le |\xi| \le 2\}$. One of the main results is the following assertion.

Theorem. *Let $0 < p < 1$, $0 < q, r \le \infty$, and $s > n(1/p - 1/2)$. Suppose that $q = r = 2$ or*

$$\frac{1-\theta}{q} + \frac{\theta}{r} < \frac{1}{2}, \quad \theta = \frac{n}{s}\Big(\frac{1}{p} - \frac{1}{2}\Big).$$

If, in addition,

$$\sup_{\delta>0} \|m(\delta\,\cdot)\eta\|^{1-\theta}_{L_q} \|m(\delta\,\cdot)\eta\|^{\theta}_{B^s_{r,\infty}} < \infty, \tag{1}$$

then m is a Fourier multiplier in $H_p(\mathbb{R}^n)$. If $(1-\theta)/q + \theta/r > 1/2$, then such an assertion cannot be valid.

Similar results hold for multipliers in the spaces $H_1(\mathbb{R}^n)$ and $L_p(\mathbb{R}^n)$, $1 \le p < 2$. For example, if (1) is satisfied for $1 \le p < 2$, $q = \infty$, $0 < r \le \infty$, and $s > n/\min(r, 2)$, then m is a Fourier multiplier in $H_p(\mathbb{R}^n)$.

Some results of this kind for multipliers in the space $L_1(\mathbb{R}^n)$ (or $L_\infty(\mathbb{R}^n)$) are obtained in [1] and [2].

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Generalized solutions to systems of Hamilton—Jacobi equations

Consider the system of n hyperbolic quasilinear equations

$$\frac{\partial u}{\partial t} + H_x(u) = 0,\ u(0,x) = u_0(x), \tag{1}$$

where $x \in R$, $t \in [0,T]$, $u(\cdot,\cdot) : [0,T] \times R \to R^n$, $H(\cdot) : R^n \to R^n$. We suppose that the initial function u_0 has small total variation, the coordinates of the function $H(\cdot)$ are continuously differentiable. It is known [1], that the solution $u(\cdot,\cdot)$ exists and it is unique in the class of functions with boundary variations.

Consider the system of n Hamilton—Jacobi equations

$$\frac{\partial v}{\partial t} + H(v_x) = 0,\ v(0,x) = \int_0^x u_0(x)dx. \tag{2}$$

We suggest a notion of generalized solution $v(\cdot,\cdot)$ for problem (2) and construct the solution $v(\cdot,\cdot)$, using the solution $u(\cdot,\cdot)$.

Consider the system of two Hamilton—Jacobi equations

$$\frac{\partial \varphi}{\partial t} + f(\varphi_x) = 0,\ \frac{\partial \psi}{\partial t} + \psi_x g(\varphi_x) = 0$$

with initial functions $\varphi(0,x) = \varphi_0(x)$, $\psi(0,x) = \psi_0(x)$. We assume that $x \in R$, $t \in [0,T]$, functions $f(\cdot)$, $g(\cdot)$ are differentiable and has sublinear growth relative to φ_x. Function $g(\cdot)$ is non decrease, functions $\varphi_0(\cdot), \psi_0(\cdot)$ are continuously differentiable. We introduce a definition of generalized solution of this system using the notion of multivalued M-solution, suggested by A.I. Subbotin [2].

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Approximation of the Riemann-Liouville derivative using the moments of a function and its applications

There are different approaches in solving problems involving fractional derivatives. The aim of this talk is to present a technique which is based on an approximation of fractional derivatives. More precisely, we shall use a series expansion containing only a function and its moments that suitably approximate the left Riemann-Liouville fractional derivative of that function, which has been introduced in [1]. Although expansion formula for fractional derivatives is not unique, this particular one has the benefit of requiring minimal regularity conditions of the function itself. After adapting the expansion formula we apply it to several problems in which fractional derivatives appear. We derive a new form of the well-known fractional integration by parts formula, and simplify the proof of one important fractional integral inequality. Moreover, we discuss the novelties achieved with the use of the expansion formula in fractional variational problems.

This talk is based on joint work with Teodor M. Atanacković, Marko Janev, Stevan Pilipović and Dušan Zorica.

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Weighted Ditzian-Totik moduli revisited

We introduce new weighted moduli of smoothness and discuss their properties (equivalence to certain K-functionals, weighted Ditzian-Totik moduli, direct and converse theorems for polynomial approximation involving these moduli, etc.). (This is a joint work with D. Leviatan and I. Shevchuk.)

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Refinement of the solution to the contact problem for the thin biphasic cartilage layers

Biomechanical contact problems involving transmission of the forces across biological joints are considerable practical importance in surgery. The model for the axisymmetric problem of contact interaction of articular cartilage surfaces was proposed in [1]. In our report, we propose a refinement of the solution to above said model. It is crucial that we assume that the contact domain has an elliptic shape. It is allowed us to get asymptotic representation of the solution in terms of a small parameter

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Ridgelet transform and asymptotic behavior of distributions

In this paper we prove that the ridgelet transform $\mathcal{R} : \mathcal{S}_0(\mathbb{R}^n) \to \mathcal{S}(\mathbb{Y}^{n+1})$ and its adjoint $\mathcal{R}^* : \mathcal{S}(\mathbb{Y}^{n+1}) \to \mathcal{S}_0(\mathbb{R}^n)$ are continuous. We extend the definition of ridgelet transform to the space od tempered distributions. Also, we provide some Abelian and Tauberian type results relating the quasyasymtotic behaviour of tempered distributions with the asymtotic behaviour of their ridgelet transform.

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Retinal Pulse Wave Velocity Assessment for in-vivo Estimation of Retinal Arterial Stiffness

Background: Human blood vessels become stiffer because of the combined effects of aging, high blood pressure, and other factors. These alterations occur primarily in small vessels. Using principles of dynamic retinal imaging, mathematical signal analysis and measurements of arterial stiffness in the macrocirculation we have recently proposed a novel diagnostic methodology. A functional clinical parameter, retinal pulse-wave velocity (rPWV), is measured in-vivo, which presumably allows to characterize arterial stiffness in the retinal microcirculation. We demonstrated previously that both aging with not excluded cardiovascular risk factors and mild arterial hypertension are associated with elevated rPWV. Whether rPWV increases with age in medically validated healthy subjects is investigated.

Methods: 71 healthy 41.0 ± 12.1 (range: $20 - 66$) years old volunteers were examined. The following cardiovascular risk factors were excluded: overweight, increased blood pressure, cholesterol level and blood glucose. Time dependent alterations of vessel diameter were measured by the Retinal Vessel Analyzer (IMEDOS Systems, Jena, Germany) in a retinal artery. The data was filtered and evaluated using mathematical signal analysis and rPWVs were calculated.

Results: rPWV amounted to 370 ± 100 (range: $180 - 620$) RU(relative units)/s in the whole group. 1RU corresponds to 1μm in the Gullstrands eye model. rPWV increased with age:

$r = 0.41$ (Pearsons correlation, $p < 0.005$). There was a weak negative correlation with vessel diameter: $r = -0.27$, $p < 0.05$ as well as a weak positive correlation with mean arterial pressure: $r = 0.22$, $p < 0.05$.

Conclusions: Healthy aging with excluded cardiovascular risk factors is associated with increased rPWV and hence with age-related elevation of retinal arterial stiffness. Nevertheless rPWV increases in medically validated healthy volunteers to a much less extent than in hypertensive subjects or in subjects with not excluded cardiovascular risk.

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Leray's problem on D-solutions to the stationary Navier-Stokes equations past an obstacle

Consider the stationary Navier-Stokes equations in the 3-dimensional exterior domain Ω. For every $u_\infty \neq 0$ and every $f \in \dot{H}_2^{-1}(\Omega)$, Leray [1] constructed a D-solution u with $\nabla u \in L_2(\Omega)$ and $u - u_\infty \in L_6(\Omega)$. Here $\dot{H}_2^{-1}(\Omega)$ denotes the dual space of the homogeneous Sobolev space $\dot{H}_2^1(\Omega)$. We prove that every D-solution necessarily satisfies the generalized energy equality. Moreover, we obtain a sharp a priori estimate and uniqueness result for D-solutions assuming only that $\|f\|_{\dot{H}_2^{-1}(\Omega)}$ and $|u_\infty|$ are suitably small. Our results give final affirmative answers to open questions proposed by Leray on the energy equality and uniqueness of D-solutions. Finally, we investigate the convergence of weak solutions as $u_\infty \to 0$ in the strong norm topology, while the limiting weak solution exhibits a completely different behavior from that in the case $u_\infty \neq 0$.

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Properties of DEs in Clifford Algebras

In the talk, we establish a number of elementary properties about the topology and algebra of real quadratic homogeneous mapping, Ricatti type ODEs and Dirac equations occurring in Clifford Algebras. We construct a series of examples of square root closed algebras with Clifford algebra as a kernel and prove that the algebras in question do necessarily contain the complex structure.

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Construction of transmutation operators and pseudoanalytic functions of a hyperbolic variable

The notion of a transmutation (transformation) operator relating two linear differential operators was introduced in 1938 by J. Delsarte and nowadays represents a widely used tool in the theory of linear differential equations (see [1], [6], [7] and references therein).

The talk is dedicated to a new representation for integral kernels of transmutation operators obtained as functional series with exact formulae for their terms. It is based on the application of hyperbolic pseudoanalytic function theory [3], [4] and recent results on mapping properties of the transmutation operators [2].

The kernel K_1 of the transmutation operator relating $A = -\frac{d^2}{dx^2} + q_1(x)$ and $B = -\frac{d^2}{dx^2}$ results to be a complex component of a bicomplex hyperbolic pseudoanalytic function satisfying a special Vekua-type hyperbolic equation. The other component of that function is the kernel K_2 of the transmutation operator relating $C = -\frac{d^2}{dx^2} + q_2(x)$ and B where q_2 is obtained from q_1 by a Darboux transformation [5]. We prove the expansion and a Runge-type theorems for this hyperbolic Vekua equation and obtain the new representation for the kernels K_1 and K_2. Moreover, based on the presented theory an approach for numerical computation of the transmutation kernels is proposed and applied to solution of spectral problems with remarkable results.

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3D geomechanical model of rock massif in the region of tectonic breaches intersection

3D geological modeling in region with large amount of tectonic breaches and following geomechanical model creation are considered in this paper. Source data handling process is a basis for model creation. Source data in our case: geological maps and seismic profiles. These data were processed to export them to the database, containing information for each profile and surface section in X,Y,Z format. Then information about each interested surface was exported to the geoinformational system MapManager [1], and interpolation process was done here. 3D-surfaces of interested breaches and horizons are output at this step.
Next step: joining and cutting of obtained surfaces was carried out to obtain qualitative geological model. Geomechanical model with presence of 8 main faults was built on the base of obtained geological model. The stress-strain state of rock massive caused by gravity was calculated and analyzed. It was stated that stress-strain state of massif with faults has essential differences as compared with stress-strain state of massive without faults.
Also such important stage before modeling as the availability the opportunity to control and

check the wells data, intervals of mineral deposits, the seismic data in the aggregate, including visualization ability (3D viewing) is described. The possibilities of implementing such control are examined on specific examples with analyzing of these situations, also concrete actions and recommendations are proposed.

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Generalized Jacobi inversion problem and application.

Let us consider classical Jacobi inversion problem $\sum \varsigma(q_\nu) \equiv q_\mu - k_\mu$ (mod of the periods), $\mu = 1, ...h$, where all notation has been taken from [1] and the number of terms in the sum exactly equals to h. This classical problem is already solved by different means. The most interesting of them to our mind is the way of constructing Riemann θ-function of a special kind and looking for its zeros. This problem has a lot of application as well as its so called real analogue, where only the real part of the system of congruences modulo of periods is used. It must be pointed out its significance in so called "singular case" of Riemann-Hilbert and Hilbert problems for multiply connected domain and Riemann surface with the edge. All known generalizations beforehand has been concerned with the variations of the number of the terms in the sum in the left part of the congruence.

Last time there appears some generalizations of the Jacobi inversion problem (see, f.i. [2]), where the system of congruences modulo uses not whole periods but some their parts (f.i. half of the periods). In some cases it could be solved by standard means (Riemann θ-function of a special kind). It means that classical Jacobi inversion problem now could be generalized by the mentioned above way.

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An overview of changes connected with elements of calculus in the syllabus of secondary schools in Poland in years 1970–2013.

This talk is about introducing the elements of calculus during a course of mathematics in secondary schools in Poland. Over the years, elements of calculus were successively eliminated from the syllabus. I will talk about the changes that have taken place over the last 40 years.

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Nonlinear Wave Equations with Strong Dissipation and Proliferation

In this talk we consider initial-Neumann boundary value problem of nonlinear evolution equations with strong dissipation and proliferation arising in mathematical biology formulated as

$$(NE)\begin{cases} u_{tt} = D\nabla^2 u_t + \nabla\cdot(\chi(u_t, e^{-u})e^{-u}\nabla u) + \mu_1 u_t(1-u_t) \\ \qquad\qquad\qquad\qquad\qquad in \;\; (x,t)\in\mathbf{\Omega}\times(0,\infty) & (1.1) \\ \dfrac{\partial}{\partial\nu}u|_{\partial\mathbf{\Omega}} = 0 \qquad\qquad on \;\; \partial\mathbf{\Omega}\times(0,\infty) & (1.2) \\ u(x,0)=u_0(x), u_t(x,0)=u_1(x) \qquad in \;\; \mathbf{\Omega} & (1.3) \end{cases}$$

where constants D, μ_1 are positive, $\mathbf{\Omega}$ is a bounded domain in R^n with a smooth boundary $\partial\mathbf{\Omega}$ and ν is the outer unit normal vector. We show the existence and asymptotic behavior of the solution. Under some conditions of the coefficient $\chi(u_t, e^{-u})$ of (1.1), we can derive the energy esitmate of (NE), which enables us to show the global existence in time of the soluton and asymptotic behaviour. In the case of $\mu_1 = 0$, we already dealt with the problem in [2] and we could improve the result in this talk. We apply our result to mathematical models of tumour invasion with proliferation of tumour cells(cf. [1]).

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General fixed point and attractor theorems and their applications

In 1990, S. Prieß-Crampe published an ultrametric version of Banach's Fixed Point Theorem which added to our understanding of important facts in valuation theory, such as Hensel's Lemma. In collaboration with P. Ribenboim, she generalized the result to ultrametric spaces with partially ordered value sets, and also introduced so-called "attractor theorems". In [1] I proved a version of the attractor theorem which allows us to derive different forms of Hensel's Lemma in several important settings. This development once again showed that extracting the underlying principle of important proofs widens the scope of the possible applications. In a next step along this way, driven by studies in the area of real algebra, I developed in collaboration with Katarzyna Kuhlmann a very basic fixed point theorem that works in a minimal setting, not involving any metrics or ultrametrics. This theorem not only covers the metric and ultrametric cases, but also generates new fixed point theorems for quasi-compact topological spaces and for ordered fields. I will describe the basic ideas together with some of the applications, and the connections with other theorems such as the Knaster-Tarski Fixed Point Theorem. If time permits, I will also sketch how this basic way of thinking, based on the comparison of the metric with the ultrametric world, may inform our search for suitable notions of "fractal" and "iterated function system" in non-metrizable settings.

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General fixed point theorems and spaces of real places

During my joint work with Franz-Viktor Kuhlmann on the spaces of real places of an ordered field it turned out that already one of the "simplest" such spaces has a very intriguing structure with lots of self-homeomorphisms (see [3]). This led us to the question whether some sort of fractality is around here. But to study it, one commonly uses fixed point theorems. Unfortunately, the spaces we study are not necessarily metric, and their natural topology is not so easy to handle. As Franz-Viktor has worked with ultrametric fixed point theorems (see [1]), we wondered what tools could be carried over to the case under consideration. This led us back to a 15 year old question: is there a generalization of the ultrametric fixed point theorems in a more topology-like language, that may serve as a common ground for both the metric and the ultrametric world? The answer, given in [2], is yes (and is surprisingly simple). I will report on a very basic fixed point theorem that works in a minimal setting, not involving any metrics. This theorem not only covers the metric and ultrametric, but also generates a new topological fixed point theorem. The approach also lays a basis for the possible adaptation of the notion of "fractal" to non-metrizable settings.

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Geometric regularization on Riemannian and Lorentzian manifolds

We investigate regularizations of distributional sections of vector bundles by means of nets of smooth sections that preserve the main regularity properties of the original distributions (singular support, wavefront set, Sobolev regularity). The underlying regularization mechanism is based on functional calculus of elliptic operators with finite speed of propagation with respect to a complete Riemannian metric. As an application we consider the interplay between the wave equation on a Lorentzian manifold and corresponding Riemannian regularizations. We also show that the restriction to underlying space-like foliations behaves well with respect to these regularizations.

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Analysis of changes in the size and distribution of the composite's reinforcing phase after plastic deformation

The aim of this study was to explain the influence of the friction stir processing (FSP) on the size and distribution of reinforcement particles in the composite F3K.10S/SiC_p, as well as determine its mechanical properties [1]-[4]. Moreover presents applications of a new theory of the representative volume element (RVE) based on the Mityushev-Eisenstein-Rayleigh sums (M-sums) [5] to describe particle-reinforced composites [6]. In the resulting FSP modification observed significant changes in the distribution of SiC particles, precipitation which was characterized by macro-heterogeneity. However, in micro scale, a few typical distributions and areas of fragmentation of the reinforcing particles were identified. Differences in the sizes of the ceramic particles were 15 microns in the initial material and a few micron in the modified material, respectively. The analysis of these regions using the new RVE theory shows that the coefficient of mechanical properties anisotropy was changed an order of magnitude. The article confirms that the Mityushev's new RVE theory resolves the problem of the constructive pure geometrical description of the properties of composites. Mechanical testing of selected parts of modified composite showed significant differences in the values of the plastic flow stress in advancing, retreating sides and weld nugget.

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On the Oscillatory Property of a Two Terms Linear Differential Equation

We consider the equation

$$l[y] = -y'' + \upsilon(x)y - u(x)y = 0 \ \ (x > 0) \qquad (1)$$

with continuous non-negative functions υ and u defined on $I = [0, \infty)$. For example we can take $\upsilon = q_+ = \max\{q, 0\}$, $u = q_- = \min\{-q, 0\}$, if q is a potential alternating in any interval (t, ∞) as $t \to \infty$. We are interested in oscillatory properties of equation (1) at infinity. For the case $\upsilon = 0$ the following oscillation condition is well known:

(2) $$\lim_{x\to\infty} \sup x \int\limits_x^{\infty} u(t)dt > 1$$

(see [1]). In [2] another oscillation condition was proved:

$$\lim_{x\to\infty} \inf A(x|\, u) > 1,$$

where $A(x|\, u) = x^{-1} \int_x^{\infty} t^2 u(t)dt + x \int_x^{\infty} u(t)dt$. Let $\mathcal{H}$ be a set of all positive continuous functions $h(x)$ $(x > 0)$ such that $\lim\limits_{x\to\infty} \inf x^{-1} h(x) \geq 1$.

Theorem. Let for $h \in \mathcal{H}$ the condition

$$\lim_{x\to\infty} \sup x \int\limits_x^{x+h(x)} (u - \upsilon)\, dt > 1 + \lim_{x\to\infty} \sup A(x|\, \upsilon)$$

be satisfied. Then equation (1) is oscillatory at infinity.

Example. The equation $-y'' - u(x)y = 0$ is oscillatory if

(3) $$\lim_{x\to\infty} \sup x \int\limits_x^{x+h(x)} u(t)dt > 1,$$

where $h(x) = \sup \left\{ d > 0 : \, d \int_x^{x+d} \upsilon(t)dt \leq 1 \right\} < \infty$, and $\upsilon(t) = \eta(1)$ if $0 < t \leq 1$, $\upsilon(\mathrm{t}) = t^{-2}\eta(\mathrm{t})$ if $t > 1$; $\eta(t) = \int_t^{\infty} u(x)dx$. Condition (3) is equivalent to condition (2) for all u such that $\lim\limits_{x\to\infty} \sup x \int_{h(x)}^{\infty} u(t)dt = 0$. Analogous results were obtained for high order two terms equations of type (1).

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Conductivity of fibrous materials with perfect fillers and holes.

The problem of conductivity of fibrous materials with fillers and holes was reduced in the article [1] to the Hilbert problem for some special multiply connected domain, fully solved in general case in [2, 3].

The present talks involves the use of known technique for investigation of the conductivity of fibrous materials with two fillers and/or holes which are the perfect circles. Some interesting practical results were computed.

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On the generalisation of the equations for self-similar profiles of semilinear wave equations.

Some results on the generalisation of the equations for self-similar profiles of semiliniear wave equations will be given. Possible extensions of the equations in which the methods described in [1] apply will be presented.

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A refinement of the Boole-Stein-Weiss theorem on the Hilbert transform and related topics

The L^p norm of the Hilbert transform of the characteristic function of a set of finite measure is invariant with respect to the structure of the set: it can be written as an explicit function of its Lebesgue measure and of the exponent p. We show that more is true: there is a fixed ratio, only dependent on p, between the L^p norms of such a Hilbert transform computed on the given set and on the whole real line. We then discuss some putative connection of this result with some open problems regarding the Hilbert transform and other linear operators related to it.

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Stability for higher order elliptic operators subject to homogeneous boundary conditions on varying domains

We consider elliptic partial differential operators of second and higher order, subject to homogeneous boundary conditions on bounded domains of the N-dimensional Euclidean space. We discuss a general theorem ensuring their spectral stability upon perturbation of the underlying domain, in the frame of the so-called E-compact convergence. Applications to the case of the bi-harmonic operator with Dirichlet, Neumann and Intermediate boundary conditions will be considered.

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Rank-M Frame Multipliers and Optimality Criterions for Density Operators of Rank M

Ever since the introduction of frames in 1952 [1], the connection between frame theory and decompositions of certain operators, particularly the identity operator, into rank-ones began to be elaborated. Abandoning the idea of restricting to tight frame-like expansions, with respect to systems arising from a single template function, one is led to the concept of resolutions of the identity, with respect to more general systems than the usual rank-one expansions of the identity. By replacing the arising rank-ones, indexed by some measure space (X, μ), with suitable positive rank-M operators over the Hilbert space $\mathcal{H}$ of the form

$$P_x := \sum_{i=1}^{M} \lambda_i \left\langle \bullet, \eta_x^i \right\rangle \eta_x^i \ , \qquad \lambda \in \mathbb{R}^M, \ x \in X \ \text{ and } \ \eta_x^i \in \mathcal{H} \ \forall \ i, x,$$

which still retains the (weak) decomposition property of the identity, $\int_X P_x \ \mathrm{d}\mu(x) = \mathbf{1}_{\mathcal{H}}$, we gain more flexibility regarding operator decompositions, without loosing too much of the well-established theory. In this study, we will investigate various notions of possible generalizations of optimality criterions for rank-M frames and corresponding multipliers of the form

$$f \ \longmapsto \int_X m(x) \ P_x f \ \mathrm{d}\mu(x) \ , \qquad \text{with } \ P_x f = \sum_{i=1}^{M} \lambda_i \left\langle f, \eta_x^i \right\rangle \eta_x^i \ \text{ as above.}$$

Explicitly, we will lay stress on continuous M-frames, arising from irreducible group representations of locally compact groups and try to find adequate notions of optimality.

Note that this will be connected to (multi-window) Weyl-Heisenberg frames and the theory of Gabor multipliers [2], pseudo-differential operators and Berezin-Toeplitz quantization [3, 4], as well as certain generalizations of these.

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A quasi-linear heat transmission problem in a periodic dilute two-phase composite. A functional analytic approach

We consider a temperature transmission problem for a composite material which fills the Euclidean space. The composite has a periodic structure and consists of two materials. In each periodicity cell one material occupies an inclusion of size ϵ, and the second material fills the remaining part of the cell. We assume that the thermal conductivities of the materials depend nonlinearly upon the temperature. We show that for ϵ small enough the problem has a solution, *i.e.*, a pair of functions which determine the temperature distribution in the two materials. Then we analyze the behaviour of such a solution as ϵ approaches 0 by an approach which is alternative to those of asymptotic analysis.

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Branching in Hermitian Clifford Analysis

Clifford analysis is a higher dimensional generalization of complex analysis and it studies solutions of the Dirac operator in the Euclidean space $\mathbb{R}^m$. Recently Gelfand-Tsetlin bases have been explicitly constructed for solutions of certain systems of invariant differential equations which are dealt with in Clifford analysis. As for numerical applications, the most important fact is that expansions into Gelfand-Tsetlin basis elements are both orthogonal and of Taylor type. Namely, all coefficients of the orthogonal expansion of a given solution are determined in terms of its partial derivatives.

Hermitian Clifford analysis is obtained by adding a complex structure to the setting in $\mathbb{R}^{2n}$ and it studies solutions of the homogeneous system of two hermitian Dirac operators. In the hermitian case, so far a construction of Gelfand-Tsetlin bases based on the Cauchy-Kovalevskaya extension has been described and explicit formulæ for basis elements have been known only in complex dimension $n = 2$, see [1, 2]. In this talk, we express explicitly the Gelfand-Tsetlin bases using Jacobi polynomials in any complex dimension, see [4]. To do this, it is sufficient to describe the so-called branching in this case, as is explained in [3].

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Gelfand-Shilov type spaces through Hermite expansions

Gelfand-Shilov spaces $\mathcal{S}_\alpha(\mathbb{R}^d), \mathcal{S}^\beta(\mathbb{R}^d)$ and $\mathcal{S}_\alpha^\beta(\mathbb{R}^d)$ and their generalisations, the Gelfand-Shilov spaces of Roumieau and Beuerling type $\mathcal{S}^{\{m\}}(\mathbb{R}^d)$ respectivly $\mathcal{S}^{(m)}(\mathbb{R}^d)$ are discussed in [2], [3], [4] and [6].In this talk we focus on the special cases $\mathcal{S}_\alpha^\beta(\mathbb{R}^d)$ (resp. $\sum_\alpha^\beta(\mathbb{R}^d)$). We show that if it converges in the sense of $\mathcal{S}_\alpha^\beta(\mathbb{R}^d)$ (resp. $\sum_\alpha^\beta(\mathbb{R}^d)$), then it belongs to $\mathcal{S}_\alpha^\alpha(\mathbb{R}^d)$ (resp. $\sum_\alpha^\alpha(\mathbb{R}^d)$).Furthermore we analyze intermideate spaces $\left(\mathcal{S}_\alpha^\alpha \otimes \mathcal{S}_\beta^\beta\right)(\mathbb{R}^{s+t})$ (resp. $\left(\Sigma_\alpha^\alpha \otimes \Sigma_\beta^\beta\right)(\mathbb{R}^{s+t})$), introduced also by Gelfand and Shilov, through the estimates of Hermite coefficients. In the last part of the talk we introduce one more class of Gelfand-Shilov type spaces $S_\sigma^{\otimes,\sigma}(\mathbb{R}^n), \sigma \geq 1/2$, and $\Sigma_\sigma^{\otimes,\sigma}(\mathbb{R}^n), \sigma > 1/2$. These spaces were obtained through the iteration of Harmonic oscilators and are related to our study of Weyl formula for tensorised products of elliptic Shubin type operators ([1], [5]). We compare all the considered spaces through the estimates of Hermite coefficients.

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On the coupling of function theoretic methods and the finite element method for a boundary value problem with singularity

In this talk we will present a continuation of a research in the direction of coupling between analytical and finite element solutions for a boundary value problem from the theory of linear elasticity with a singularity. The modelled problem will be a domain containing a crack with the crack tip. We will construct the analytical solution in the crack tip region and couple it with a FE solution for the part of domain which is free of singularities.

The analytical solution of the crack-tip problem is based on the *Formulae of Kolosov*, and represented by two holomorphic functions $\Phi(z)$ and $\Psi(z)$, $z \in \mathbb{C}$ [5]. The solution based on the complex function theory gives us a high accuracy of the solution in the neighbourhood of the singularity. The disadvantage of the complex analytic approach is that the full linear elastic boundary value problem can be solved explicitly only for some simple or canonical domains. To solve problems for more complicated domains we combine the advantages of the both approaches.

Our idea is to continue a work in the direction proposed in [1], [2], [3] and [4] for a method of coupling between analytical and finite element solutions. The main goal of this approach is to get a continuous coupling between analytical and finite element solutions through the whole interaction interface. For that reason, we construct a special element that contains an exact solution to the differential equation with the correct singularity and coupling elements.

Finally, ideas for the generalization to the three-dimensional case by means of quaternionic analysis will be presented.

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Fundamental groups over p-adic analytic spaces

A naive definition of analytic spaces over non-archimedean fields gives rise to totally disconnected spaces. Tate solved this problem by restricting the coverings one should consider, thus defining a coarser Grothendieck topology. Another approach, giving rise to topological spaces (and not simply Grothendieck topologies) is given by Berkovich in [Ber90] by considering new points, corresponding to evaluation of functions in bigger nonarchimedean fields, or equivalently corresponding to multiplicative seminorms on the algebra of power series.

In the algebraic setting, Grothendieck defines finite étale covers giving rise to a profinite fundamental group, which, when the base field is $\mathbb{C}$, is the profinite completion of the fundamental group of the corresponding analytic space [Gro71]. However, in Berkovich geometry, the analytification of an algebraic étale cover is not a topological cover. One can however define various categories of (infinite) étale covers over Berkovich space, giving rise to various fundamental groups ([2],[1]). We will study them, in the case of smooth curves over $\mathbb{C}_p$.

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Chaos expansion method for the Skorokhod integral equation

The Skorokhod integral equation in the framework of chaos expansion methods on white noise probability space is considered. A necessary and sufficient condition for existence of the solution is proposed. Moreover, the solution is expressed in explicit form. The integral equation is transformed into an equivalent system of two equations. The first involves the Ornstein-Uhlenbeck operator and the second one is a differential equation involving the Malliavin derivative operator. The chaos expansion method is applied in order to solve the first equation. The general solution belongs to the space of Kondratiev generalized stochastic processes. In addition, the Gaussian processes and Gaussian solutions of the equation are described in detail.
(Work in colaboration with Dora Seleši and Stevan Pilipović)

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Audio Inpainting using M-Frames

Classical short-time Fourier constructions [1] lead to a signal decomposition with a fixed time-frequency resolution. This is due to the fact that only one unique window is used for analysis. However, having signals with varying features, for example audio signals, such time-frequency decompositions can be very restrictive.

One possibility to achieve a more flexible and adaptive sampling of the time-frequency plane is called nonstationary Gabor transform and was recently introduced in [2]. The resolution can evolve over time or frequency, respectively, by using different windows for the different sampling positions in the time or frequency domain. Obviously, this construction will then lead to a multi-window frame (M-frame). Decomposing an audio signal using such frames can provide a sparser representation in the time-frequency domain than regular single window frames.

One possible application could be to exploit the sparsity of M-frame decompositions for audio inpainting. Audio inpainting tries to recover some audio signal x which is depleted with a degradation matrix A, i.e., a specific number of coefficients of x are set to zero. The inpainting problem can be formulated as a convex minimization problem

$$\arg\min_x \|Ax - b\|_2^2 + \tau \|\Psi(x)\|_1, \tag{1}$$

where b is the depleted signal, Ψ an linear operator transforming x to a sparse representation and where τ is some constraining parameter. This problem can be solved using the forward backward algorithm from [3] or the Douglas-Rachford algorithm presented in [4].

In this talk, we would like to present numerical results for different choices of the linear operator Ψ. Starting from a classical single window Gabor frame (Ψ would then be the classical Gabor transform), we compare the results with several multi window frames, where the focus is mainly on the reconstruction error and not on the runtime. Possible multi window frames are nonstationary Gabor frames and wavelet frames, which result from a special case of nonstationary Gabor frames, where the resolution changes over frequency, i.e., the analyzing windows have varying bandwidths.

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Fourier transform versus Hilbert transform

We present several results in which the interplay between the Fourier transform and the Hilbert transform is of special form and importance.

1. In 50-s, the following problem in Fourier Analysis attracted much attention: *Let $\{a_k\}_{k=0}^{\infty}$ be the sequence of the Fourier coefficients of the absolutely convergent sine (cosine) Fourier series of a function $f : \mathbb{T} = [-\pi, \pi) \to \mathbb{C}$, that is $\sum |a_k| < \infty$. Under which conditions on $\{a_k\}$ the re-expansion of $f(t)$ ($f(t) - f(0)$, respectively) in the cosine (sine) Fourier series will also be* *absolutely convergent?*

We solve a similar problem for functions on the whole axis and their Fourier transforms. Generally, the re-expansion of a function with integrable cosine (sine) Fourier transform in the sine (cosine) Fourier transform is integrable if and only if not only the initial Fourier transform is integrable but also the Hilbert transform of the initial Fourier transform is integrable.

2. The following result is due to Hardy and Littlewood: *If a (periodic) function f and its conjugate $\widetilde{f}$ are both of bounded variation, their Fourier series converge absolutely.*

We generalize the Hardy-Littlewood theorem (joint work with U. Stadtmüller) to functions on the real axis and their Fourier transforms. The initial Hardy-Littlewood theorem is a partial case of this extension, when the function is taken to be of compact support.

3. These and other problems are closely related to the behavior of the Fourier transform of a function of bounded variation. We have found the maximal space for the integrability of such a Fourier transform. Along with those known earlier, various interesting new spaces appear in this study. Their inter-relations lead, in particular, to improvements of Hardy's inequality.

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Functions of bounded Λ-variation and integral smoothness

We obtain the necessary and sufficient condition for embeddings of integral Lipschitz classes $\text{Lip}(\alpha; p)$ into spaces ΛBV of functions of bounded Λ-variation. This answers a question of Wang [2].

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Localization Operators for Ridgelet Transforms

We prove that localization operators associated to ridgelet transforms with L^p symbols are bounded linear operators on $L^2(\mathbb{R}^n)$. Operators closely related to these localization operators are shown to be in the trace class and a trace formula for them is given.

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Toeplitz operators on the harmonic Bergman space with pseudo differential defining symbols

In [3] A. Sánchez-Nungaray and N. Vasilevski studied the C^*-algebra generated by Toeplitz operators acting on the Bergman over the unit disk $\mathbb{D}$ on the complex plane, whose pseudo-differential defining symbols belong to the algebra $\mathcal{R} = \mathcal{R}(C(\overline{\mathbb{D}}); S_{\mathbb{D}}, S_{\mathbb{D}}^*)$. The last algebra is generated by the multiplication operators $a(z)I$, where $a(z) \in C(\overline{\mathbb{D}})$, and the following two singular integral (pseudodifferential) operators

$$(S_{\mathbb{D}}\varphi)(z) = -\frac{1}{\pi}\int_{\mathbb{D}} \frac{\varphi(\zeta)}{(\zeta - z)^2} d\nu(\zeta) \quad \text{and} \quad (S_{\mathbb{D}}^*\varphi)(z) = -\frac{1}{\pi}\int_{\mathbb{D}} \frac{\varphi(\zeta)}{(\overline{\zeta} - \overline{z})^2} d\nu(\zeta).$$

It turns out that, for the Bergman space case, both algebras $\mathcal{T}(C(\overline{\mathbb{D}}))$, which is generated by Toeplitz operators T_a with defining symbols $a(z) \in C(\overline{\mathbb{D}})$, and $\mathcal{T}(\mathcal{R}(C(\overline{\mathbb{D}}); S_{\mathbb{D}}, S_{\mathbb{D}}^*))$, which is generated by Toeplitz operators T_A with defining symbols $A \in \mathcal{R}(C(\overline{\mathbb{D}}); S_{\mathbb{D}}, S_{\mathbb{D}}^*)$, are, in fact, the same; and the Fredholm symbol algebra for both of them is isomorphic and isometric to $C(\mathbb{T})$, where $\mathbb{T}$ is the boundary of the unit disk $\mathbb{D}$.

In this talk we prove that, if we consider Toeplitz operators acting on the harmonic Bergman space, the C^*-algebra generated by all Toeplitz operators T_A with $A \in \mathcal{R}(C(\overline{\mathbb{D}}); S_{\mathbb{D}}, S_{\mathbb{D}}^*)$ is not isomorphic to the C^*-algebra generated by all Toeplitz operators T_a with $a \in C(\overline{\mathbb{D}})$.

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Differential Equations with degenerated variable operator at the derivative

The theory of Jordan chains for multiparameter operator-functions $A(\lambda) : E_1 \to E_2$, $\lambda \in \Lambda$, $\dim\Lambda = k$, $\dim E_1 = \dim E_2 = n$ is developed. Here $A_0 = A(0)$ is degenerated operator, $dim Ker A_0 = 1$, $Ker A_0 = \{\varphi\}$, $Ker \mathrm{A}_0^* = \{\psi\}$ and the operator-function $A(\lambda)$ is supposed to be linear on λ. Applications to degenerate differential equations of the form $[A_0 + R(\cdot, x)]x' = Bx$ are given.

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On C∗-Algebras of Toeplitz Operators on the Harmonic Bergman Space

In this talk we present some recent results on the study of C^*-algebras generated by Toeplitz operators acting on the harmonic Bergman space on the unit disk. We describe three different algebras of Toeplitz operators acting on the harmonic Bergman space: The C^*-algebra generated by Toeplitz operators with radial symbols, in the elliptic case; the C^*-algebra generated by Toeplitz operators with piecewise continuous symbols, in the parabolic and hyperbolic cases. We prove that the Calkin algebras of the first two algebras are commutative, like in the case of the Bergman space, while the last one is not.

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Fundamental solutions of a singular heat conduction operator with a singularity on the coordinate hyperplane

Let J_p be Bessel function of the first kind and $j_p(t) = \frac{2^p\,\Gamma(p+1)}{t^p}\,J_p(t)$. Let us introduce kernels of integral transformation

$$\Lambda^{\pm}_{\gamma}(x',\xi') = \prod_{i=1}^{n} j_{\frac{\gamma_i-1}{2}}(x_i\xi_i) \mp i\,\frac{x_i\xi_i}{\gamma_i+1}\; j_{\frac{\gamma_i+1}{2}}(x_i\xi_i)\,.$$

The mixed full direct and inverse Fourier-Bessel transformations (reversibility and some properties of this integral transformation see in [1]) are defined by the formulas, respectively

$$F_B[f(x)](\xi) = \int\limits_{-\infty}^{\infty} f(x)\,\Lambda^{+}_{\gamma}(x\xi)\,e^{-i\langle x'',\xi''\rangle}\;(x^2)^{\frac{\gamma}{2}}\,dx,$$

$$F_B^{-1}[f(\xi)](x)=\frac{1}{2^{\gamma}\,\Gamma^2\left(\frac{\gamma+1}{2}\right)}\int\limits_{-\infty}^{\infty} f(\xi)\,\Lambda^{-}_{\gamma}(x\xi)\,e^{-i\langle x'',\xi''\rangle}\;(\xi^2)^{\frac{\gamma}{2}}\,d\xi\,,$$

The function $\frac{r}{x_1}$, $r = \sqrt{\sum_{i=1}^{N} x_i^2}$ has a singularity at $|x'| \neq 0$, $x' = (x_1,\dots,x_{i-1},x_{i+1},\dots,x_N)$ and $x_1 \to 0$. Thus, the singularity of this function is concentrated on the part of the coordinate hyperplane $x_1 = 0, |x'| > 0$.

We introduce a singular differential operator

$$N_B = \frac{\partial}{\partial t} - a^2\Delta_B + b\frac{r}{x_1}\,\frac{\partial}{\partial x_1}, \qquad (1)$$

where $\Delta_B = \sum_{i=1}^{n} B_i + \sum_{j=n+1}^{N} \frac{\partial^2}{\partial x_j}$, $\quad B_i = \frac{\partial^2}{\partial x_i^2} + \frac{\gamma_i}{x_i}\frac{\partial}{\partial x_i}$ (see [2], p.20, (1.2.5)).

The fundamental solution of the singular differential operator (1) is solved by full direct and inverse Fourier-Bessel transformations. It is the function

$$E(x,t)=\frac{t^{-\frac{N+\gamma}{2}}\ e^{-\frac{(|x|-\mathbf{b}t)^2}{4a^2t}}}{2^{N+\gamma-n}\ \pi^{\frac{N-n}{2}}\ \prod_{i=1}^{n}\Gamma\left(\frac{\gamma_i+1}{2}\right)}.$$

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Analytic solutions of heat type equations

We give an extension of the mean-value property and its converse to the case of real analytic functions and to functions of Laplacian growth. As an application we give a characterization of analytic solutions in time variable of the initial value problem to the heat equation $\partial_t u = \Delta u$ in terms of holomorphic properties of the solid and/or spherical means of the initial data. Next study heat type equations $\partial_t u = \widetilde{\Delta} u$, where the operator $\widetilde{\Delta}$ is given by a sum of squares of commuting, real analytic vector fields acting on a real analytic manifold. We give necessary and sufficient conditions for convergence and Borel summability of formal power series solutions in terms of generalized integral means of the initial data.

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The use of variables in a patterning activity

The use of variables is really significant in secondary education, when the students are expected to be able to create, understand and manipulate symbolic expressions. A 'patterning approach' has been proposed as a fruitful way to introduce the notion of variable; a patterning activity may begin with an exploration by the students, followed by a discussion and comparisons which are expected to lead them to a general rule. The results of relevant studies have shown students' difficulties in generalising patterns in an algebraic form (English and Warren, 1998; Orton and Orton, 1999). We organised a teaching experiment in order to examine how Polish secondary school students perceive the notion of variable in a patterning activity. Our main research question was: What are the different uses of variables by students during their engagement in a patterning task?

A worksheet was delivered to the students, who firstly had to answer some questions and then make a short presentation about their findings. In our analysis we firstly located all the instances when there was explicit reference on a variable. Then we analysed the utterances in order to identify the meanings assigned to the variables. Finally, we analysed the progress of each group by examining and comparing the utterances used throughout the lesson. Our data have led us to two categories: in the first the variable was treated as a generalized number, while in the second it was not treated as such; in that category we have distinguished three subcategories, in which the variable was: (a) closely linked to the referred object, (b) used in a superfluous manner and (c) treated as a constant. It is important to note that in most cases the students have shown a move between these categories, especially from the second to the first one.

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Structure of Cesàro function spaces and interpolation

The *Cesàro function spaces* $Ces_p(I)$ on both $I = [0,1]$ and $I = [0,\infty)$ are the classes of Lebesgue measurable real functions f on I such that the norm $\|f\|_{C(p)} = \left[\int_I \left(\frac{1}{x}\int_0^x |f(t)|dt\right)^p \, dx\right]^{1/p} < \infty$ for $1 \le p < \infty$ and $\|f\|_{C(\infty)} = \sup_{x\in I,\ x>0} \frac{1}{x}\int_0^x |f(t)|\, dt < \infty$ for $p = \infty$. In the case $1 < p < \infty$ spaces $Ces_p(I)$ are separable, strictly convex and not symmetric. They, in the contrast to the sequence spaces, are not reflexive and do not have the fixed point property.

The structure of the Cesàro function spaces $Ces_p(I)$ is investigated. Their dual spaces, which equivalent norms have different description on $[0,1]$ and $[0,\infty)$, are described. The spaces $Ces_p(I)$ for $1 < p < \infty$ are not isomorphic to any $L^q(I)$ space with $1 \le q \le \infty$. They have "near zero" complemented subspaces isomorphic to l^p and "in the middle" contain an asymptotically isometric copy of l^1 and also a copy of $L^1[0,1]$. They do not have Dunford-Pettis property. Cesàro function spaces on $[0,1]$ and $[0,\infty)$ are isomorphic for $1 < p < \infty$. Moreover, the Rademacher functions span in $Ces_p[0,1]$ for $1 \le p < \infty$ a space which is isomorphic to l^2. This subspace is uncomplemented in $Ces_p[0,1]$. The span in the space $Ces_\infty[0,1]$ gives another sequence space.

In [4] and [5] it was shown that $Ces_p(I)$ is an interpolation space between $Ces_{p_0}(I)$ and $Ces_{p_1}(I)$ for $1 < p_0 < p_1 \le \infty$ for $1/p = (1-\theta)/p_0 + \theta/p_1$ with $0 < \theta < 1$. The same result is true for Cesàro sequence spaces. On the other hand, $Ces_p[0,1]$ is not an interpolation space between $Ces_1[0,1]$ and $Ces_\infty[0,1]$.

The talk is based on joint papers with Sergey V. Astashkin.

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Interpolation problems in the classes of entire functions of zero order

We consider the interpolation problem in the class E_0 of entire functions of zero order: $F(a_n) = b_n$, $n = 1, 2, \dots$, where the sequence $A = \{a_n\}$ has limit point ∞, and the numbers $\{b_n\}$ satisfy the condition

$$\limsup_{n\to\infty} \frac{\ln^+ \ln^+ |b_n|}{\ln |a_n|} \le 0\,.$$

Two criteria of resolvability of this problem are received. The first criterion is formulated by the canonical product of the interpolation knots, the second criterion is formulated by the measure generated by these knots.

The following result is valid.

Theorem. *Let* $\{a_n\}_{n\in\mathbb{N}}$, $a_i \ne a_j$ $(i \ne j)$, *be the sequence of points which has limit point* ∞. *The following three statements are equivalent*:

(1) *the sequence* A *is an interpolation sequence in the class* E_0;

(2) *for all* $\varepsilon > 0$ *the following relation is true*:

$$\sum_{n=1}^{\infty} \frac{1}{|a_n|^{\varepsilon}} < \infty\,, \tag{1}$$

and canonical product $E_A(z) = \prod_{n=1}^{\infty} \left(1 - \frac{z}{a_n}\right)$, *of* A *satisfies the condition*:

$$\lim_{n\to\infty} \frac{1}{\ln |a_n|} \ln^+ \ln \frac{1}{|E'_A(a_n)|} = 0\,;$$

(3) (1) *is true and*

$$\lim_{z\to\infty} \frac{1}{\ln |z|} \ln^+ \int_0^1 \frac{(n_A(C(z, \alpha|z|)) - 1)^+}{\alpha}\, d\alpha = 0\,.$$

Here n_A – counting function of quantity of points of A.

The research was carried out on a subject No 0111U002152.

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Local approximation on arcs in the complex plane

Besides the classical theory of polynomial approximation such as Jackson and Bernstein theorems defining global approximation, Nikolsky, Timan and Dzyadyk theorems determining local-global approximation, there exist purely local polynomial approximations. The first such theorem as well-known was obtained by P.M.Tamrazov and V.V.Berdzinskiy. They were considered a class of function on $[1, -1]$ interval defined in $x_0 \in [1, -1]$ point in which $|f(x) - f(x_0)| \le$

$const|x-x_0|$. They proved that there exists a such polynomial $P_n(x)$ for which $|f(x_0)-P_n(x_0)| \leq \frac{const}{n^\alpha}$.

On solving the problem of approximation on arcs we introduced a local class of functions $D_\alpha^\beta(z_0,\Gamma)$ ($z_0 \in \Gamma$, Γ - arbitrary arc in a complex plane) ($0 < \alpha \leq 1$, $\beta \geq 0$), for which $\forall z_1, z_2$ $|f(z_1) - f(z_2)| \leq C(\Gamma) max\{|z_0 - z_1|^\beta, |z_0 - z_2|^\beta\}, |z_1 - z_2|^\alpha$.

In addition, we considered also local class of functions introduced by A.A.Gonchar $H_\alpha^{\alpha+\beta}(z_0,\Gamma)$ consisting of functions $f(z) \in H^\alpha(\Gamma)$ (class of Holder α order), and for which $|f(z) - f(z_0)| \leq const|z - z_0|^{\alpha+\beta}$ ($0 < \alpha \leq 1$, $\beta \geq 0$). For these classes of functions, we obtained direct and inverse theorems of polynomial approximations, closer each other. That allows us to get the constructive characterization for classes of $D_\alpha^\beta(z_0,\Gamma)$ and $H_\alpha^{\alpha+\beta}(z_0,\Gamma)$.

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A continuous-time generalization of Motzkin's theorem of the alternative

A theorem of the alternative (or a transposition theorem) asserts that two systems are related in such a way that exactly one of them is consistent. These theorems play a important role in deriving optimality conditions for wide classes of extremal problems. We will present some generalizations of well known Motzkin's theorem of the alternative, i.e., we focus on Motzkin's theorem that represents a continuous-time analogue of the original theorem. Moreover, the presented approach can be used in finite dimensional case also, and we will present some new results pertaining to (in)consistency of systems described with strict convex inequalities and affine equalities.

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On generalizations of quasicontinuity and of cliquishness of multi-value maps

Let X, Y and Z be topological spaces. For a map $f : X\times Y \to Z$ and a point $p = (x, y) \in X\times Y$ put $f^x(y) = f_y(x) = f(p)$. The symbol $C(f)$ denotes the set of points of continuity of a map f. Recall that a multi-valued map $F : X \to Y$ is said to be *lower quasicontinuous at a point* $x_0 \in X$ if for any neighbourhood U of $x_0 \in X$ and any open set V in Y with $F(x_0) \cap V \neq \emptyset$ there exists a nonempty open set G in X such that $G \subseteq U$ and $F(x) \cap V \neq \emptyset$ for any $x \in G$. Denote by $K^-(F)$ the set of all points at which the map F is lower quasicontinuous. If $K^-(F) = X$ then F is said to be *lower quasicontinuous*.

Theorem 1. *A multi-value map $F : X \to Y$ is lower quasicontinuous if and only if for any nonempty open set U in X and for any set $A \subseteq X$ such that $U \subseteq \overline{A}$ we have that $F(U) \subseteq \overline{F(A)}$.*

A multi-value map $F : X \to Y$ we call *lower pseudo-quasicontinuous* if for any nonempty open set U in X and for any set $A \subseteq X$ with $U \subseteq \overline{A}$ there exists a nonempty open set G in X such that $G \subseteq U$ and $F(G) \subseteq \overline{F(A)}$.

Theorem 2. *Let $F : X \to Y$ a multi-value map such that $X \setminus K^-(F)$ is nowhere dense. Then F is lower pseudo-quasicontinuous.*

A multi-value map $F : X \to Y$ is called *lower covering categorical cliquish* if for every open cover $\mathcal{V}$ of the space Y and any non-meagre set E in X there is a somewhere dense set $A \subseteq X$ and a set $V \in \mathcal{V}$ such that $A \subseteq E$ and $F(x) \cap V \neq \emptyset$ for all $x \in A$. If Y is a second countable space then every multi-value map $F : X \to Y$ is lower covering categorical cliquish.

Theorem 3. *Let $F : X \to Y$ a multi-value map such that $K^-(F)$ is a residual set in X. Then F is lower covering categorical cliquish.*

Using the notions of lower pseudo-quasicontinuity and lower covering categorical cliquishness we generalize the results of [1,2].

Theorem 4. *Let X and Y be topological spaces, Z a metrizable space, $\mathcal{V} = \{V_n : n \in \mathbb{N}\}$ a countable system of sets in Y, $\mathcal{V}_y = \{V \in \mathcal{V} : V -$ is neighborhood of y in $Y\}$, $B(\mathcal{V}) = \{y \in Y : \mathcal{V}_y -$ is base neighborhood of y in $Y\}$, $f : X \times Y \to Z$ a map such that for each set $V \in \mathcal{V}$ with $V \cap B(\mathcal{V}) \neq \emptyset$ a map $F_V : X \ni x \mapsto F_V(x) = f^x(V) \subseteq Z$ is both lower pseudo-quasicontinuous and lower covering categorical cliquish. Then $R = \{x \in X : \{x\} \times (C(f^x) \cap B(\mathcal{V})) \subseteq C(f)\}$ is a residual subset of X.*

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B_σ-function spaces and Calderón-Zygmund operators

In [1, 2], in order to unify the central Morrey spaces, the λ-central mean oscillation spaces and the usual Morrey-Campanato spaces, we introduced B_σ-function spaces $B_\sigma(E)(\mathbb{R}^n)$, $0 \le \sigma < \infty$, and unified a series of results on the boundedness for integral operators on several classical function spaces. Here $B_\sigma(E)(\mathbb{R}^n)$ is the set of all functions f on $\mathbb{R}^n$ such that

$$\|f\|_{B_\sigma(E)} = \sup_{r \ge 1} \frac{1}{r^\sigma} \|f\|_{E(Q_r)} < \infty,$$

where $Q_r = \{y = (y_1, y_2, \cdots, y_n) \in \mathbb{R}^n : \max_{1 \le i \le n} |y_i| < r\}$ or $Q_r = \{y \in \mathbb{R}^n : |y| < r\}$, and $E(Q_r)$ is a function space on Q_r with semi norm $\| \cdot \|_{E(Q_r)}$.

Let $E = L^p$. If $\sigma = n/p$, then $B_\sigma(L^p)(\mathbb{R}^n) = B^p(\mathbb{R}^n)$ which introduced by Beurling (1964) together with its predual $A^p(\mathbb{R}^n)$, so-called the Beurling algebra.

Let $E = L_{p,\lambda}$ (Morrey space) or $\mathcal{L}_{p,\lambda}$ (Campanato space). If $\sigma = 0$, then $B_\sigma(L_{p,\lambda})(\mathbb{R}^n)$ and $B_\sigma(\mathcal{L}_{p,\lambda})(\mathbb{R}^n)$ coincide $L_{p,\lambda}(\mathbb{R}^n)$ and $\mathcal{L}_{p,\lambda}(\mathbb{R}^n)$, respectively. Moreover, $B_\sigma(L_{p,\lambda})(\mathbb{R}^n)$ unifies $L_{p,\lambda}(\mathbb{R}^n)$ and $B^{p,\lambda}(\mathbb{R}^n)$ (non-homogeneous central Morrey space), and $B_\sigma(\mathcal{L}_{p,\lambda})(\mathbb{R}^n)$ unifies $\mathcal{L}_{p,\lambda}(\mathbb{R}^n)$ and $\mathrm{CMO}^{p,\lambda}(\mathbb{R}^n)$ (λ-central mean oscillation space).

And also using B_σ-function spaces, we can study both local and global regularities of functions simultaneously.

In this talk, we consider the boundedness for Calderón-Zygmund operators, especially commutators $[b, T]$ generated by $b \in L^1_{loc}(\mathbb{R}^n)$ and Calderón-Zygmund operator T, i.e.,

$$[b, T]f(x) = b(x)Tf(x) - T(bf)(x), \quad x \in \mathbb{R}^n,$$

on the B_σ-function spaces.

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Dirichlet's Problem by Using Computers with the Theory of Reproducing Kernels

We shall give practical and numerical solutions of the Laplace equation on multidimensional spaces and show their numerical experiments by using computers. Our method is based on the Dirichlet principle by combinations with generalized inverses, Tikhonov's regularization and the theory of reproducing kernels.

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On the LQR/LQG Design Problem and the Associated Riccati Differential Equations

The numerical treatment of linear quadratic regulator/gaussian design problems for parabolic partial differential equations requires solving large-scale Riccati equations. In the finite time horizon case, the Riccati differential equation (RDE) arises. We show that within a Galerkin projection framework the solutions of the finite-dimensional RDEs converge in the strong operator topology to the solutions of the infinite-dimensional RDEs. Moreover, we briefly review efficient numerical methods for solving RDEs capable of exploiting the structure of the involved coefficient matrices (e.g. sparse, symmetric, low rank).

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On Asymptotic Analysis of Localised Boundary-Domain Integral Operators for Variable-Coefficient PDEs.

The localized boundary-domain integral equation (LBDIE) systems introduced in [1] and associated with the Dirichlet and Neumann boundary value problems (BVP) for a scalar "Laplace" PDE with *variable* coefficient are considered. The parametrix is localized by multiplication with

a radial localizing function. Mapping and jump properties of the surface and volume integral potentials based on a localized parametrix and constituting the LBDIE systems were studied in the Sobolev (Bessel potential) spaces in [2] and the LBDIEs equivalence to the original variable-coefficient BVPs and the invertibility of the LBDIE operators in the corresponding Sobolev spaces were proved.

In this contribution we discuss how the norms of the LBDIE operators depend on the scaling parameter of the localising function, when it tends to zero. Although the considered LBDIE operators are invertible for any size of the characteristic domain of localization, it appears that the norms of the operators and of their inverse can grow as the size decreases, for some of the LBDIEs. This effect may be responsible for the deterioration of convergence of the mesh-based and mesh-less numerical methods of LBDIE solution, cf. [3], [4], observed for fine meshes (large number of collocation points).

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Examples of ill-posedness for the Euler and the quasi-geostrophic equations

I will describe local ill-posedness of solutions of the 2D quasi-geostrophic and the 3D Euler equations in the Besov and the logarithmic Lipschitz spaces.

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$\mathbb{R}$–linear problem and its applications

Let D_k be mutually disjoint simply connected domains bounded by smooth curves ∂D_k and D be the complement of all closures of D_k to the torus T^2. To find a function $\varphi(z)$ analytic in $D^+ = D_1 \cup \ldots \cup D_n$, $D^- = D$ and continuous in the closures of the considered domains on the torus T^2 with the following conjugation condition $\varphi^+(t) = \varphi^-(t) - \rho\overline{\varphi^-(t)}$, $t \in \partial D$, where ρ is a constant. Application of the generalized method of Schwarz yields a constructive algorithm to solve the problem. Relations to the Riemann–Hilbert problem and applications to conformal mappings and to composite media are discussed.

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Spectral and scattering prblems on star-shaped graphs with nontrivial compact parts

We consider Schrödinger operators on star-shaped graphs with nontrivial compact parts such as finite rays and/or a loop. Under suitable decay conditions on the potential we obtain a complete form of the spectral representation and apply it to the time dependent formulation of the scattering theory.

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An estimation method of shift parameters in image separation problem

In this talk, we focus on the simplest spatio-temporal mixing model of blind source separation [1] for images. Here, an image means $T = (T_1, T_2) \in \mathbb{R}^2$ periodic bounded real-valued function $f(t)$ of $t \in \mathbb{R}^2$. Let N be the number of sources and J, $J \geq N$, be the number of observed images. In the simplest spatio-temporal mixing model, we assume that observed images $x_j(t)$, $j = 1, \dots, J$, are the following mixtures of original source images $s_n(t)$, $n = 1, \dots, N$:

$$x_j(t) = \sum_{n=1}^{N} d_{j,n}\, s_n(t - c_{j,n}),$$

where $d_{j,n} \in \mathbb{R}$ are mixing coefficients and $c_{j,n} \in \mathbb{R}^2$ are shift parameters. For each fixed $j \neq 1$, we assume that $c_{j,n} - c_{1,n} \neq c_{j,k} - c_{1,k}$ when $n \neq k,\ n, k = 1, \dots, N$. In the case of blind source separation, we only know the observed images. Then we want to estimate the number of sources N, model parameters $c_{j,n}$ and $d_{j,n}$, and separate the original source images.

For image separation without shift [2], we proposed a new image separation method using annular sector multiwavelet functions $\psi_p(t)$, $p = 1, \dots, P$. Since each Fourier transform of ψ_p is a compactly supported smooth function, we can define the continuous multiwavelet transform of image f with respect to ψ_p as

$$W_{\psi_p} f(b, a) = \frac{1}{a} \int_{\mathbb{R}^2} f(t)\, \overline{\psi_p\left(\frac{t-b}{a}\right)} dt,$$

where scale parameter $a > 0$ and position parameter $b \in \mathbb{R}^2$. For each a, $W_{\psi_p} f(b, a)$ is a T periodic complex-valued function of b.

We propose an estimation method of shift parameters $c_{j,n}$ using the following complex-valued correlation functions $R^{1,j}_{\psi_p}(t, a)$ between $W_{\psi_p} x_1(b, a)$ and $W_{\psi_p} x_j(b, a)$ defined by

$$R^{1,j}_{\psi_p}(t, a) = \int_{[0,T_1)\times[0,T_2)} W_{\psi_p} x_1(b, a) \overline{W_{\psi_p} x_j(t - b, a)}\, db.$$

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Toeplitz operators and boundary value problems of wave diffraction by gratings

In the past few years we made several attempts to find explicit inverses for a class of convolution type operators issued from wave diffraction by periodic gratings. Starting with an operator formulation of the boundary value problem, and using operator equivalence relations, we end up with a Toeplitz operator acting between spaces defined on a composed contour. Then we consider generalized factorizations of the corresponding Fourier symbols in the setting of L^2 spaces on the contour.

First, we obtained an inverse for a particular geometry of the grating and given Neumann conditions on the boundary [1], afterwards we discussed the Fredholmness and invertibility conditions for a more general geometry of the grating and other boundary conditions [2].

Now, we are able to present an approximate result based on a Neumann series for the inverse operator, where the explicit generalized factorization is obtained for the Fourier symbols of the Neumann and the oblique derivative boundary value problems that holds for small wave numbers.

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On existence of the resolvent and discreetness of the spectrum of a class of differential operators of hyperbolic type

We consider in the space $L_2(\Omega)$ the differential operator of hyperbolic type

$$L_0 u = u_{xx} - u_{yy} + a(y)\, u_x + c(y)\, u$$

with the domain $D(L_0)$ of infinitely differentiable functions satisfying the conditions $u(-\pi;\, y) = u(\pi;\, y)\,,\ \ u_x(-\pi;\, y) = u_x(\pi;\, y)$ and compactly supported with respect to the variable y, where

$$\Omega = \{(x,y):\, -\pi < x < \pi,\ -\infty < y < \infty\}\,.$$

Further, we assume that the coefficients $a(y)\,,\ c(y)$ satisfy the conditions: *i)* $|a(y)| \geq \delta_0 > 0,\ \ c(y) \geq \delta > 0$ are continuous functions in $R = (-\infty;\, \infty)$. It is easy to verify that the operator L_0 admits closure in the space $L_2(\Omega)$, which is denoted by L.

Theorem 1. Let the condition $i)$ be fulfilled. Then the operator $L + \mu E$ is continuously invertible for $\mu \geq 0$.

Theorem 2. Let the condition $i)$ be fulfilled. Then the resolvent of the operator L is compact if and only if for any $w > 0$

$$\lim_{|y|\to\infty} \int_y^{y+w} c(t)\, dt = \infty. \qquad (*)$$

The question of the existence of the resolvent and discrete spectrum in an unbounded domain with growing and oscillating coefficients was previously studied only in the case of elliptic and pseudodifferential operators [1-3].

Assume that the coefficients of the operator L, in addition to conditions $i)$, satisfy the condition

$$ii)\ \mu_0 = \sup_{|y-t|\leq 1} \frac{c\,(y)}{c\,(t)} < \infty,\ \ \mu = \sup_{|y-t|\leq 1} \frac{a\,(y)}{a\,(t)} < \infty.$$

Theorem 3. Let the conditions $i)$-$ii)$ be fulfilled. Then the resolvent of the operator L is compact if and only if $\lim\limits_{|y|\to\infty} c\,(y) = \infty$.

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Correction Theorems

The paper is related to fixed point theorem in nonsmooth analysis. In the classical analysis the fixed point theorem is widely used in the mathematics and has practical applications, in particular, in mechanics. Paper's results are based on generalization of Kakutani's fixed theorem [1] in case of multiple-valued matrix nonconvex mapping. Kakutani's generalized fixed point theorem allows solve many problems in nonsmooth analysis, including research of nonsmooth implicit functions and correction theorem. For the first time the correction theorem was solved by H.Halkin for smooth systems [2], for nonsmooth systems - by V.F. Demyanov [3]. Three types of correction theorems (in smooth and nonsmooth cases) are considered.

Let $f : \mathbb{R}^n \to \mathbb{R}^m,\ m \leq n,\ f = (f_1, \ldots, f_m)$, f_i - smooth functions on open set $S \subset \mathbb{R}^n$, $x \in S$. For all i one has

$$f_i(x+\Delta) = f_i(x) + (f_i'(x), \Delta) + o_i(\Delta), \quad \frac{o_i(\Delta)}{\|\Delta\|} \underset{\|\Delta\to 0\|}{\to} 0.$$

It is possible to formulate the following types of the correction theorem.

The first type of correction theorem. Find vector-function $\nu_1(\Delta)$ such that

$$f_i(x+\Delta+\nu_1(\Delta)) = f_i(x) + (f_i'(x), \Delta) \qquad \forall i \in 1:m, \quad \frac{\nu_1(\Delta)}{\|\Delta\|} \underset{\|\Delta\to 0\|}{\to} 0_n.$$

The second type of correction theorem. Find vector-function $\nu_2(\Delta)$ such that

$$f_i(x+\Delta) = f_i(x) + (f_i'(x), \Delta+\nu_2(\Delta)) \qquad \forall i \in 1:m, \quad \frac{\nu_2(\Delta)}{\|\Delta\|} \underset{\|\Delta\to 0\|}{\to} 0_n.$$

The third type of correction theorem. Find vector-function $\nu_3(\Delta)$ such that

$$f_i(x+\Delta) = f_i(x+\nu_3(\Delta)) + (f_i'(x+\nu_3(\Delta)), \Delta) \qquad \forall i \in 1:m, \quad \frac{\nu_3(\Delta)}{\|\Delta\|} \underset{\|\Delta\to 0}{\to} 0_n.$$

If there exists function $\nu_i(\Delta) (i \in 1:3)$, then correction problem of the i type has solution, and the function $\nu_i(\Delta)$ is correction of the i type of the function f in neighborhood of x.

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Effective conductivity of a dilute periodic two-phase composite with imperfect thermal contact at the two-phase interface

We consider the effective thermal conductivity of a two-phase composite with imperfect thermal contact at the two-phase interface. The composite is obtained by introducing into an infinite homogeneous matrix a periodic set of inclusions of a different material. The diameter of each inclusion is assumed to be proportional to a positive real parameter ϵ. Then we show that the function which describes the effective conductivity can be continued real analytically in the parameter ϵ around the value $\epsilon = 0$ (in correspondence of which the inclusions collapse to points). The methods developed are based on functional analysis and potential theory and are alternative to asymptotic analysis.

The talk is based on [1, 2].

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Energy solutions for nonlinear Klein-Gordon equations in de Sitter spacetime

The Cauchy problem for nonlinear Klein-Gordon equations is considered in de Sitter spacetime. The nonlinear terms are power type or exponential type. The local and global solutions are shown in the energy class.

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Asymptotic behavior of solutions to damped wave equation with derivative nonlinear term

In this talk we study the Cauchy problem for damped wave equation with derivative nonlinear term:

(1) $$u_{tt} - \Delta u + u_t = |\nabla_x u|^\sigma, \quad t > 0,\ x \in \mathbb{R}^n,$$

with initial data

(2) $$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad x \in \mathbb{R}^n,$$

where $\sigma > 1 + 2/(n+1)$ is a constant. The Cauchy problem for the corresponding semilinear damprd wave equation

$$(3) \qquad u_{tt} - \Delta u + u_t = |u|^\sigma, \quad t > 0, \ x \in \mathbb{R}^n$$

has been investigated by several authors. Moreover, it is known that equation (3) admits global solution in time, when $\sigma > 1 + 2/n$ and initial data are sufficiently small.

We will show the existence in global in time and asymptotic behavior of the solution to (1)–(2), provided that $n = 1, 2, 3$ and onitial data (u_0, u_1) are sufficiently small.

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Klein-Gordon type wave models with non-effective time-dependent potential

We consider the Cauchy problem for Klein-Gordon type models,

$$u_{tt} - \Delta u + m(t)^2 u = 0, \ \ u(0, x) = u_0(x), \ \ u_t(0, x) = u_1(x),$$

with $tm(t) \to 0$, i.e., $m(t)^2 u$ is so-called non-effective time-dependent potential. We define and describe the long time behavior for some appropriate energy. A scattering result will complete our considerations.

The results presented in this talk can be found in the recent accepted paper [1].

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Fast algorithm for computing e–sums

Consider two-dimensional two-component periodic composite made from a collection of non-overlapping, identical, circular disks, embedded in a matrix. In accordance with a theory of the representative cells (representative volume elements), the effective conductivity of disks is expressed in terms of the the e–sums introduced as discrete multidimensional convolutions of the Eisenstein functions [2]. Straightforward computation of the e–sums is possible only for the sums of lower orders. In the present talk, a fast algorithm to compute higher order sums worked out by use of random walks and Monte Carlo simulations. Relations between the Eisenstein and Weierstrass functions and algebraic dependences between their derivatives are used to improve the algorithm. The obtained numerical results are applied to investigation of the structure of composites.

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On orthogonal monogenics in oblate spheroidal domains and recurrence formulae

A complete orthogonal system for the monogenic L_2-space consisting of solid oblate spheroidal monogenics $\{\Phi_n^m\}$ in $\mathbb{R}^3$ is constructed by means of harmonic functions. The lack of symmetry leads to some differences compared with the orthogonal polynomial Appell system $\{A_k^l\}$ for spherical domains. Explicit representation formulae of the obtained basis functions $\{\Phi_n^m\}$ by $\{A_k^l\}$ are given as well as recurrence formulae for fast computer implementations.

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Convolutions For the Fractional Fourier Transform and Applications

Despite the great advances which have been recently made in the theory and applications of fractional Fourier transforms, there remains much to be worked out in their associated notions and theory. This talk presents new convolutions which have powerful properties when associated with the fractional Fourier transform here considered. Namely, we will prove the convolution theorems associated with the FRFT, together with their natural algebraic properties such as commutativity, associativity and distributivity. This will be therefore an extra tool with potential use in signal processing and other types of applications. This is here illustrated by considering corresponding classes of convolution equations to which we obtain necessary and sufficient conditions for their solvability, and give their corresponding solutions in explicit form.

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The functional analytic foundation of Colombeau algebras

Colombeau algebras constitute a rigorous framework for performing nonlinear operations like multiplication on Schwartz distributions in a way suitable for application in physics. Many variants and modifications of these algebras exist for various applications. We present a functional analytic approach placing these algebras in a unifying hierarchy, which clarifies their structural properties as well as their relation to each other.

The core of our approach is to define a basic space of generalized functions consisting of smooth mappings from a space of smoothing operators to a space of smooth functions. Apart from the conceptual simplicity and universality of this approach, one can recover many classical Colombeau algebras in this setting. Furthermore, requirements for the following key properties can be clarified:

(1) existence of the so-called σ-embedding of smooth functions, which allows one to split the smooth from the singular part in calculations;
(2) sheaf properties;
(3) invariance under diffeomorphisms and derivatives;
(4) existence of a meaningful directional derivative which is C^∞-linear in the directional vector field.

The obtained results will be crucial for eventually establishing a notion of covariant derivative in an algebra of nonlinear generalized tensor fields.

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An extension of Turán's inequality for ultraspherical polynomials

The m-th ultraspherical polynomial $P_m^{(\lambda)}$, $m \in \mathbb{N}$, is orthogonal in $[-1,1]$ with respect to the weight function $w_\lambda(x) = (1-x^2)^{\lambda-1/2}$, $\lambda > -1/2$, and is normalized by $P_m^{(\lambda)}(1) = \binom{m+2\lambda-1}{m}$. Set

$$p_m(x) = p_m^{(\lambda)}(x) := P_m^{(\lambda)}(x)/P_m^{(\lambda)}(1), \quad m = 0, 1, \ldots, \tag{1}$$

where, for the sake of brevity, the superscript $^{(\lambda)}$ will be oppressed hereafter. We prove the following extension of Turán's inequality:

Theorem. *Let p_n be defined by (1), and $\lambda \in (-1/2, 1/2]$. Then, for every $n \in \mathbb{N}$,*

$$|x|p_n^2(x) - p_{n-1}(x)p_{n+1}(x) \geq 0 \quad \text{for every} \ \ x \in [-1,1]\,. \tag{2}$$

The equality in (2) holds only for $x = \pm 1$ and, if n is even, for $x = 0$. Moreover, (2) fails for every $\lambda > 1/2$ and $n \in \mathbb{N}$.

This variation of Turán's inequality was introduced by Gerhold and Kauers [1] and proven in the limit case $\lambda = 1/2$, i.e., for the Legendre polynomials. We present both analytical and computer proof of this result. An extension to Jacobi polynomials is discussed.

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The Cauchy problem for a coupled system of the damped wave equations

We consider the Cauchy problem for a coupled system of damped wave equations

$$\begin{cases} u_{tt} - \Delta u + u_t = |v|^p, \\ v_{tt} - \Delta v + v_t = |u|^q, \end{cases} \quad (t,x) \in \mathbf{R}_+ \times \mathbf{R}^N \tag{P}$$

with $(u, v, u_t, v_t)(0,x) = (u_0, v_0, u_1, v_1)(x),\ x \in \mathbf{R}^N$. It was shown for the corresponding system of heat equations by M. Escobedo and M. A. Herrero [1] that

$$\alpha := \max\Big(\frac{p+1}{pq-1}, \frac{q+1}{pq-1}\Big) = \frac{N}{2} \tag{$*$}$$

gives the critical exponents.

In my talk we discuss the relation between $(*)$ and the Fujita critical exponent $\rho_F(N) = 1 + \frac{2}{N}$ on the single equation, and moreover, seek for the asymptotic profiles of the global-in-time solution for small data in the supercritical exponent and the blow-up time of the local solution for suitable data in the subcritical exponent, which is mainly based on [2].

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Local and microlocal Cauchy problem for noneffectively hyperbolic operators

We study differential operators of order 2 and give a sufficient condition in order that the micro supports of solutions to the Cauchy problem propagate with finite speed. We then study the Cauchy problem for noneffectively hyperbolic operators with no null bicharacteristic tangent to the doubly characteristic set and with zero positive trace. Checking the sufficient condition for the propagation with finite speed, we prove that the Cauchy problem for such noneffectively hyperbolic operators is C^∞ well posed if and only if the Levi condition is verified.

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[9] Nishitani, T., *Cauchy Problem for Noneffectively Hyperbolic Operators*, to appear in Mem. Math. Soc. Japan

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A Riemannn surface approach for diffraction from rational wedges

The explicit representation of waves diffracted from non-rectangular wedges belongs to a famous class of open problems in diffraction theory. These problems are often modelled by Dirichlet or Neumann boundary value problems for the $2D$ Helmholtz equation with complex wave number. They have been analyzed before by several methods such as the Malinzhinets method using Sommerfeld integrals, the method of boundary integral equations from potential theory or Mellin transformation techniques. These approaches lead to results which are particularly useful for asymptotic and numerical treatment.

In this talk, we present a new approach where we develop new representation formulas of the solutions which are based upon the solutions to Sommerfeld diffraction problems. We make use of symmetry properties, which require a generalization of these formulas to Riemann surfaces in order to cover arbitrary rational angles of the wedge. The approach allows us to prove well-posedness in suitable Sobolev spaces and to obtain explicit solutions in closed analytic form, or, if not available, in terms of series expansions which present the unique solution with the help of a bounded linear operator acting from the space of the boundary data into a subspace of $H^1(\Omega)$ of finite energy solutions, or into other appropriate Sobolev spaces $H^{1+\epsilon}(\Omega)$. The circumstance that we restrict to rational angles, $\alpha = \pi m/n$ with $m, n \in \mathbb{N}$, is intimately related to the nature of the representations of the solutions.

■ **Craig A. Nolder** Florida State University, Tallahassee, FL, USA, craiganolder@hotmail.com,

Möbius Geometry of $\mathbb{R}^{1,1}$

The split-complex plane $\mathbb{R}^{1,1}$ compactifies as a projective torus in $\mathbb{RP}^3$. We discuss the Möbius geometry of this compactification. This includes fixed points, transitivity and properties of the cross ratio. We also discuss properties of rational functions defined of this surface.

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Numerical methods for solving inverse problems for some hyperbolical equations

The report considers inverse problems for wave end acoustic equations. Acoustic inverse problem consists in determining the functions $c(x,y), \rho(x,y)$ and $u(x,y,t)$, which satisfies the following system:

$$\begin{aligned}
&c^{-2}(x,y)u_{tt} = \Delta u - \nabla ln\rho \nabla u,\ x>0, y\in\mathbb{R}, t>0;\\
&u|_{t<0} \equiv 0\\
&u_x(+0,y,t) = g(y,t);\\
&u|_{x=0} = f(y,t), y\in\mathbb{R}, t>0.
\end{aligned}$$

Here function $c(x,y)$ describes the speed of the wave propagation, $\rho(x,y)$ is the density of the medium, $u(x,y,t)$ is exceeded pressure and $f(y,t)$ is known data on the surface.In the report we consider that function $c(x,y)$ is known. Therefore the problem reduces to determination of the function $\rho(x,y)$.
Then the approach of I.M.Gelfand, B.M. Levitan and M.G. Krein(GLK-method) is applied for solving considered inverse problem. The essence of this method lies in reduction of the nonlinear inverse problem to a one-parameter set of linear Fredholm equations of the first of second kind. In particular, acoustic inverse problem can be reduced to the following set of integral equations(in assumption that all considerable function can be represented as a finite Fouriet sum):

$$\Phi^k(x,t) = \frac{1}{2}\sum_m \int_{-x}^{x} (f_m^k)'(t-s)\Phi^m(x,s)ds - \frac{1}{2}\int_{-\pi}^{\pi}\frac{e^{iky}}{\rho(0,y)}dy, k\in\mathbb{Z}$$

Required function $\rho(x,y)$ is connected with the set of the solutions of GLK-equation $\Phi^m(x,x-0)$ and can be restored, if GLK-equation is solved. The same method can be applied for solving inverse problem for determining coefficient $q(x,y)$ of the wave equation $u_{tt}(x,y) = \Delta u(x,y) - q(x,y)u$. The possibility of using GLK-method for solving inverse problems of elasticity and seismic is considered. Also the report contains the result of numerical experiments for solving GLK-equations with different methods, such that regularization method, reduction to the linear system and Monte-Carlo method. The comparative analysis of considered methods is represented.

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Inequalities for convolution

In this talk, we study the convolution of functions on $\mathbb{R}^n$:

$$(K * f)(x) = \int_{\mathbb{R}^n} K(x-y) f(y) dy, \qquad x \in \mathbb{R}^n.$$

The following generalization of Young's inequality is due to O'Neil : if $1 < p < q < \infty$, $0 < \tau_1 \le \tau_2 \le \infty$ and $\frac{1}{r} = 1 - \frac{1}{p} + \frac{1}{q}$, then

(1) $$L^{p,\tau_1} * L^{r,\infty} \subset L^{q,\tau_2},$$

where $L^{p,\tau} = L^{p,\tau}(\mathbb{R}^n)$ is the Lorentz space.

Find sufficient conditions on weights μ and ν, so that

$$L^p(\mu) * L^{r,\infty} \subset L^q(\nu), \qquad 1 < p \le q < \infty, \quad 1 < r \le \infty.$$

Theorem 1. *Let $1 < p \le q < \infty$ and $1 < r < \infty$. Let weights μ and ν satisfy, for any $\lambda \ge 1$,*

(2) $$\mu^*(\lambda t) \lesssim \frac{\mu^*(t)}{\lambda^\alpha}, \qquad ,\nu^*(\lambda t) \lesssim \frac{\nu^*(t)}{\lambda^\beta}, \qquad t > 0,$$

for some $\alpha \ge 0$ and $\beta \ge 0$ such that

(3) $$\alpha + 1/r + 1/p > 1, \qquad \beta + 1/r + 1/q' > 1.$$

Then a sufficient condition for

(4) $$L^p(\mu^{-1}) * L^{r,\infty} \subset L^q(\nu)$$

to hold is

$$\mathcal{G} := \sup_{|E|=|W|} \frac{\nu(E)\mu(W)}{|E|^{1+\frac{1}{r}+\frac{1}{p}-\frac{1}{q}}} < \infty,$$

where the supremum is taken over all measurable sets E and W of the same positive measure. A necessary condition for (4) is

$$\mathcal{S} := \sup_{|E|=|W|} \frac{\nu(E)\mu(W)}{|E|^{1+\frac{1}{p}-\frac{1}{q}}|E-W|^{\frac{1}{r}}} < \infty,$$

where the supremum is taken over all measurable sets E and W of the same positive measure.

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A criterion for the resolvent of a Schrodinger operator with δ-interactions to be kernel

Let $c_k > 0$, $\{t_k\}_{k=1}^{\infty} \in (0,\infty) : \ t_k \nearrow \infty$,

$$q(x) = \sum_{k=1}^{\infty} c_k \delta(x - t_k).$$

Let us consider Sturm-Liouville operator L, which is generated by differential expression $l(y)$:

(1) $$l(y) = -y'' + q(x)y$$

with domain

$$D(L) = \left\{y \in W_2^1(I):\ y'(0) = 0,\ y \in W_2^2(I\backslash\{t_k\}),\ l(y) \in L_2(I),\ y'(t_k+0) - y'(t_k-0) = c_k y(t_k)\right\}.$$

Operator L is self-conjugate (see [1]).

Let us define Otelbaev function [2]

$$q^*(x) := \inf\{d^{-2} : \sum_{t_k \in \left[x-\frac{d}{2}, x+\frac{d}{2}\right]} c_k \leq d^{-1}\}.$$

Theorem 1. *The following statements are equivalence:*

1) The spectrum of operator L is discrete;

2) $q^*(x) \to \infty$ when $|x| \to \infty$;

3) For every $\Delta > 0$ the following holds $\sum_{t_k \in [y, y+\Delta]} c_k \to \infty$ for $y \to \infty$.

Theorem 2. *Let the spectrum of operator L is discrete. $L^{-1} \in \sigma_1$ if and only if*

$$\int_0^\infty \frac{dx}{(q^*(x)+1)^{1/2}} < \infty.$$

Corollary 3. *Let $c_k = 1$, $t_k = k^\alpha$ then $L^{-1} \in \sigma_1$ if and only if $0 < \alpha < 1/3$.*

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Microlocal analysis in Colombeau algebras

Algebras of generalized functions initially had been introduced to handle the problem of multiplication of distributions as it arises in nonlinear differential equations with distribution data and in perturbation expansions in quantum field theory. In the past decade it became obvious, that – especially the Colombeau algebras – would provide a framework for linear partial differential equations with non-smooth coefficients. In this situation, the problem of multiplying distributions arises from products of the non-smooth coefficients with derivatives of the, in general, equally non-smooth solutions.

Apart from the existence theory for generalized solutions, the questions of regularity and propagation of singularities are important issues (for understanding the behavior of the generalized solutions). By now, a full-fledged regularity theory, based on pseudodifferential operators, Fourier integral operators, duality theory and (generalized) microlocal analysis has been developed in joint efforts by many authors.

This talk is intended as a survey of microlocal methods in the Colombeau framework, some of their applications and future directions.

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On Hardy inequality for disfocality and disconjugacy of half–linear differential equations of second order

Notions such as disfocal and disconjugate equations play a very important role in the qualitative theory of differential equations. In theory of linear and half–linear second order differential equations disfocality and disconjugacy properties on a given interval with regular and singular endpoints have been investigated comparatively less than nonoscillatory properties of these equations. The Riccati technique and the Lyapunov, La Vallee–Poussin and Opial inequalities are often used to find disfocality and disconjugacy properties on a given interval (see e.g., [1, Chapter 5]). We investigate the problems of disfocality and disconjugacy on a given interval of half–linear second order differential equations. We obtain new disfocality and disconjugacy conditions of half–linear differential equations by applying (see e.g., [2]) the variational method and the known results of the theory of weighted Hardy type inequalities.

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Weighted inequalities for a class of matrix operators: the case $1 < q < p < \infty$

We find necessary and sufficient conditions on weighted sequences ω_i, $i = 1, 2, ...n-1$, u and v, for which the operator $(S_{n-1}f)_i = \sum_{k_1=1}^{i} \omega_{1,k_1} \cdots \sum_{k_{n-1}=1}^{k_{n-2}} \omega_{n-1,k_{n-1}} \sum_{j=1}^{k_{n-1}} f_j$, $i \geq 1$ is bounded from $l_{p,v}$ in $l_{q,u}$, for $1 < q < p < \infty$.

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Gauge theoretic methods in real algebraic geometry

I will report on a recent joint paper with A. Teleman (arXiv:1206.4271):

'A wall crossing formula for degrees of real central projections'

The main idea in this paper is that the principle of conservation of numbers in complex algebraic geometry should be replaced by wall crossing formulas in real algebraic geometry.

In my talk I will:

- introduce the concept of relative orientations of maps between not necessarily orientable manifolds
- sketch the proof of a wall crossing formula for real central projections
- describe the geometry of the wall in the space of these maps
- explain these results with examples arising from applied mathematics, namely spaces of rational functions, and Wronski maps

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Approximate dual M-frames constructions: the Gabor case

The constructions of M-frames from a template window, which is itself the superposition of several windows, thus creating a new window having special properties suitable for a given application, implies large computational costs of handling and storage for the highly redundant expansion. An alternative for easing the extensive computations would be the use of approximate dual systems. We are aiming to provide a general numerical procedure for approximate M-frames constructions and from it we will derive all the algorithmical steps needed to construct approximate multi-window dual Gabor atoms.

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The conditions of local differentiability of solutions of stationary Schrödinger equation with discontinuous coefficients

Let $n \geq 3$ and Ω be a bounded set in R^n. It is known [1], that if in the equation

$$-\Delta u + q(x)u = f(x), \quad x \in \Omega, \tag{1}$$

the functions q, f belong to the class $L_{p,loc}(\Omega)$, $p > n$, then its generalized solution u has continuous partial derivatives of the first order in Ω. Such statement is not true, if $q, f \in L_{n,loc}(\Omega)$. The question arises: if the restrictions to functions q and f in equation (1) to formulate in terms other than L_p spaces, then what are the conditions on these spaces that are sufficient to ensure that the solution u be a continuously differentiable inside Ω?

The answer to this question represents a specific theoretical interest and is important in applications. In this work it is received for the normed spaces $M(\Omega)$ with the norm $\|\cdot\|_{M(\Omega)}$, satisfying the following conditions:

a) the set $C_0^\infty(\Omega)$ is dense in $M(\Omega)$;

b) $M(\Omega) \hookrightarrow L_1(\Omega)$;

c) if $\varphi \in M(\Omega)$, then $\psi\varphi \in M(\Omega)$ for any function ψ from $C_0^\infty(\Omega)$;

d) if $\varphi \in M(\Omega)$, then $|f| \in M(\Omega)$ and $\||f|\|_{M(\Omega)} \leq C\,\|f\|_{M(\Omega)}$, where C is independent of the f.

Assume that $0 < \alpha < n$. We denote by $P_\alpha(\Omega)$ the space obtained by completing $C_0^\infty(\Omega)$ on the norm

$$\|u\|_{P_\alpha(\Omega)} = \sup_{x\in\Omega} \int\limits_\Omega \frac{|u(y)|}{|x-y|^\alpha} dy.$$

It is easy to verify that $P_\alpha(\Omega)$ satisfies all of the conditions a) - d).

Theorem. *If the functions q and f belong to the class $M(\Omega)$ with the properties a) - d), then the generalized solution u of the equation (1) is continuously differentiable inside Ω if and only if $M(\Omega) \subseteq P_{n-1}(\Omega)$.*

We note that the spaces of non-smooth functions satisfy the conditions a) - d). However, the theorem allows us to find exact conditions of differentiability of the solutions of equation (1) in terms of the spaces of smooth functions, for example, S.L. Sobolev space $W_p^s(\Omega)$ $(1 \leq p \leq \infty,\ s > 0)$ and .V. Besov space $B^s_{p,\theta}$ $(1 \leq \theta, p \leq +\infty,\ s > 0)$.

[1] Ladyzhenskaya, O. A. and Ural'tseva, N. N., *Linear and Quasilinear Elliptic Equations*, (Russian), Nauka, Moscow, (1973).

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Hardy type inequalities on balls

This talk is based on my recent joint-work with Shuji Machihara and Hidemitsu Wadade. We revisit the Hardy inequalities on balls with radius R at the origin in $\mathbb{R}^n$ with $n \geq 2$. We describe how the behavior of functions on the boundary affects the Hardy type inequalities. A special attention is paid on the case $n = 2$ with logarithmic correction.

[1] O. A. Ladyzhenskaya, *The mathematical theory of viscous incompressible flow, Second English edition, revised and enlarged. Translated from the Russian by Richard A. Silverman and John Chu. Mathematics and its Applications, Vol. 2*, Gordon and Breach, Science Publishers, New York-London-Paris, (1969).

[2] J. Leray, *Etude de diverses équations integrales non linéaires et de quelques problèmes que pose l'hydrodynamique*, J. Math. Pures Appl. **12** (1933), 1–82.

[3] S. Machihara, T. Ozawa and H. Wadade, *Hardy type inequalities on balls*, Tohoku Math. J. (in press)

■ **Dixan Peña Peña** Ghent University, Ghent, Belgium, email: dpp@cage.ugent.be

An alternative proof of the inverse Fueter mapping theorem

Fueter's theorem [3] is a basic result in Clifford analysis which provides a way to generate axial monogenic functions starting from a holomorphic function. The aim of this talk is to present an alternative proof of the fact that the Fueter mapping is surjective on the set of axial monogenic functions. The method we present here is complementary to the one obtained in [1] and can be found in [2]. As a byproduct, we also obtain an explicit description of the kernel of the Fueter mapping.

[1] F. Colombo, I. Sabadini and F. Sommen, *The inverse Fueter mapping theorem in integral form using spherical monogenics.* Israel Journal of Mathematics 2012, DOI: 10.1007/s11856-012-0090-4.
[2] F. Colombo, D. Peña Peña, I. Sabadini and F. Sommen, *A new integral formula for the inverse Fueter mapping theorem*, arXiv: 1302.0685, 2013.
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Continuous slice functional calculus in quaternionic Hilbert spaces

We define a continuous functional calculus in quaternionic Hilbert spaces, starting from basic issues regarding the notion of spherical spectrum of a normal operator. As properties of the spherical spectrum suggest, the class of continuous functions to consider in this setting is the one of slice quaternionic functions. Slice functions generalize the concept of slice regular function, which comprises power series with quaternionic coefficients on one side and that can be seen as an effective generalization to quaternions of holomorphic functions of one complex variable. The notion of slice function allows to introduce suitable classes of real, complex and quaternionic C^*–algebras and to define, on each of these C^*–algebras, a functional calculus for quaternionic normal operators. In particular, we establish several versions of the spectral map theorem.

[1] Ghiloni, R., Moretti, V. and Perotti, A., *Continuous slice functional calculus in quaternionic Hilbert spaces*, Rev. Math. Phys., **25**, No. 4 (2013).

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The Essential Boundary in Hilbert Spaces of Polyanalytic Functions.

A Fredholm symbolic calculus is constructed for poly-Toeplitz operators with continuous symbol and I will explain how such symbol requires the notion of j-essential boundary. The symbol calculus is well known for Bergman-Toeplitz operators, in which setting the removal boundary is a subset of the boundary having zero transfinite diameter. Some surprising differences between the analytical and the poly-analytical case will be presented.

[1] Luís V. Pessoa, *Toeplitz Operators and the Essential Boundary on Polyanalytic Functions*, Internat. J. of Math., **24**, No. 6 (2013) 1350042 (23 pages) (DOI: 10.1142/S0129167X13500420).

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The Gomory-Hu inequality and balleans of finite ultrametric spaces

It was proved by Gomori and Hu in 1961 that for every finite nonempty ultrametric space (X,d) the following inequality $|\mathrm{Sp}(X)| \leqslant |X|-1$ holds with $\mathrm{Sp}(X)=\{d(x,y)\colon x,y\in X,\, x\neq y\}$ (see [1]). Denote by $\mathfrak{U}$ the set of spaces for which the equality in this inequality is attained. The spaces $X\in\mathfrak{U}$ are characterized by structural properties of some graphs connected with X. It is shown that a finite ultrametric space (X,d), $|X|\geqslant 2$, belongs to $\mathfrak{U}$ if and only if the rooted tree T_X representing (X,d) is strictly binary and the labels of different internal nodes are different (see [2]).

For a metric space X we denote by $\mathbf{B}_X$ the set of all balls (ballean) of this space.

Let X and Y be metric spaces. A mapping $F\colon X\to Y$ is called *ball-preserving* if for every $Z\in\mathbf{B}_X$ and $W\in\mathbf{B}_Y$ the following relations

$$F(Z)\in\mathbf{B}_Y \text{ and } F^{-1}(W)\in\mathbf{B}_X,$$

hold, where $F(Z)$ is the image of Z under the mapping F and $F^{-1}(W)$ is the preimage of W under this mapping.

It is shown that the rooted trees T_X and T_Y representing finite ultrametric spaces X and Y are isomorphic if and only if there exists a ball-preserving bijection $F\colon X\to Y$ (see [3]).

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Optimal Polynomial Admissible Meshes for $\mathscr{C}^{1,1}$ compacta of $\mathbb{R}^d$

Admissible Meshes (**AM**) [1] are sequences $\{A_n\}_{n\in\mathbb{N}}$ of finite subsets of a given compact set $K\subset\mathbb{R}^d$ (or $\mathbb{C}^d$) such that $\mathrm{Card}(A_n)$ grows polynomially w.r.t. n and the following inequality holds for any polynomial $p\in\mathscr{P}^n(\mathbb{R}^d)$ of degree at most n

$$\|p\|_{\mathscr{C}(K)}\leq C\max_{A_n}|p|. \tag{1}$$

AM was introduced in [1] as suitable sets where to perform the sampling for the *discrete least square* polynomial approximation. In [2] A. Kroó defines **Optimal AMs** as AMs which cardinality grows at optimal (i.e. $\sim\dim\mathscr{P}^n(\mathbb{R}^d)$) rate $\mathcal{O}(n^d)$. He also provides an existence result for star-like smooth compact subsets of $\mathbb{R}^d$.

In this talk we present our recent result [3][Th. 3.7].

Theorem 1. *If $\Omega\subset\mathbb{R}^d$ is a bounded $\mathscr{C}^{1,1}$ domain, then $\overline{\Omega}$ has an Optimal Admissible Mesh.*

The proof is fully constructive and lays on the regularity properties that the *distance function* (w.r.t. $\complement\Omega$) inherit from the boundary.

The main features of AMs are also discussed, both as theoretical motivations and applications. Namely, starting from an AM, one can extract by standard Linear Algebra some quasi-optimal interpolation arrays, say *Approximate Fekete* and *Leja points* [4]. Moreover the sequence of uniform probability measures $\{\mu_n\}_{\mathbb{N}}\subset\mathscr{M}(K)$ canonically associated to an AM $A_n\subset K$ can be used to compute the *Pluripotential Equilibrium Measure* μ_K of the given compact K and the *Siciak Extremal Function* V_K. Finally given an holomorphic function f the sequence of $L^2_{\mu_n}$-least squares polynomials p_n is *maximally convergent* to f on K (see [5]).

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Evolutionary Equations with Material Laws Containing Fractional Integrals

A well-posedness strategy for a time-shift invariant class of evolutionary operator equations as discussed in [1, 2] is applied to material laws involving fractional time-integration. This leads to evolutionary problems with fractional time-derivatives. Fractional time-differentiation is established in the framework of an function calculus for the time derivative ∂_0 as a normal operator. A class of such material laws of the form

$$\sum_{\alpha\in\Pi} \partial_0^{-\alpha} M_\alpha$$

with bounded, linear coefficient operator $M_\alpha : H \to H$, $\alpha \in \Pi$, H Hilbert space, where Π is a finite subset of $[0, 1]$, is characterized, for which well-posedness of the corresponding evolutionary problem

$$\left(\sum_{\alpha\in\Pi} \partial_0^{1-\alpha} M_\alpha + A\right) U = f,$$

where for example A is skew-selfadjoint in H, can be shown. The approach is exemplified by an application to a fractional Kelvin-Voigt type model in solid mechanics.

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Regularity properties of generalized functions

Stevan Pilipovic

We introduce new spaces of Zygmund and Bessov - type generalized functions. Then we consider Schwartz's distributions embedded into such soaces and find conditions which imply that an embedded distribution is actually an embedded Zygmund or Besov function. With this investigations we complete our program of determining spaces of generalized functions which correspond to classical function spaces.

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Monogenic functions in infinite-dimensional vector spaces with a commutative multiplication

Analytic function methods in the complex plane for plane potential fields inspire searching analogous effective methods for solving spatial problems of mathematical physics.

An algebraic-analytic approach to equations of mathematical physics is developed at the Department of Complex Analysis and Potential Theory of the Institute of Mathematics of the National Academy of Sciences of Ukraine. This approach means a finding of commutative Banach algebra such that monogenic (continuous differentiable in the sense of Gateaux) functions with values in this algebra have components satisfying the given equation with partial derivatives. Such algebras are constructed for the biharmonic equation and the three-dimensional Laplace equation and elliptic equations degenerating on an axis that describe axial-symmetric potential fields (see [1, 2, 3, 4]).

We consider monogenic functions taking values in topological vector spaces being expansions of certain infinite-dimensional commutative Banach algebras associated with the three-dimensional Laplace equation. We establish that every harmonic function is a component of the mentioned monogenic functions (see [5]).

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In memory of Promarz M. Tamrazov

Promarz M. Tamrazov (1933 – 2012) was an outstanding and leading specialist in complex analysis, potential theory and related fields of Mathematics.

He solved many open problems which were posed and tackled by other scientists. In particular:

- he developed the theory of complex finite-difference smoothnesses of any order on general sets in the complex plane and solved the difference contour-solid problems for holomorphic functions posed by Sewell in 1942, and developed a general contour-solid theory for holomorphic and meromorphic and subharmonic functions. The obtained results enabled to solve open problems of approximation theory on complex sets (see [1, 2, 3]);

- he solved the Gonchar's extremal problem on capacities of condensers and for this purpose he developed a method based on mixing signed measures or charges (see [4]);

- he investigated general properties of extremal lengths and extremal metrics, and solved problems concerning finding extremal metrics and moduli of some nonorientable and twisted Riemannian manifolds, including the problem for Mobius strip that had been tackled by Pu in

1952 but not solved (see [5, 6]);

- he solved extremal problems for conformal mappings associated with multipole quadratic differentials (see [7, 8]).

His fundamental results gave rise to fruitful investigations of many mathematicians.

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Berkovich spaces over $\mathbb{Z}$

In the late eighties, Vladimir G. Berkovich defined a notion of p-adic analytic space that has since developed quickly and found numerous applications (see [1]). Although those spaces usually appear in a non-archimedean setting, it is worth noticing that their general definition actually allows arbitrary Banach rings as base rings, e.g. $\mathbb{Z}$ endowed with the usual absolute value. Over the latter, they look like fibrations that contain complex analytic spaces as well as p-adic analytic spaces for every prime number p.

In [2], we carried out a detailed study of the affine analytic line over $\mathbb{Z}$. The purpose of the talk is to investigate the local theory of affine spaces of any dimensions. More precisely, we will explain that it is possible to generalize the Weierstrass division theorem to this setting and then follow quite closely the strategy that is used in complex analytic geometry.

We obtain the following results (see [3]). Let x be a point in an affine analytic space over $\mathbb{Z}$. We prove that the local ring $\mathcal{O}_x$ is Noetherian, regular and excellent. A direct study ensures that it is also Henselian. Pushing the methods further, we show that the structure sheaf $\mathcal{O}$ is coherent. The results actually also hold for more general base rings such as rings of integers of number fields or discrete valuation rings (with the additional assumption that their fraction fields have characteristic 0 as regards excellence).

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On the position of a chaotic solution close to a homoclinic orbit

The assumption that the critical point of a homoclinic solution is a fixed point with respect to a perturbation was used in [2] to investigate the persistence of a homoclinic trajectory in discontinuous systems with the critical point on the discontinuity boundary. This assumption creates also new possibilities for investigation of solutions close to the homoclinic orbit of unperturbed problem. We apply a construction of a solution of perturbed discontinuous system via fixed point theorem proposed by Battelli and Fečkan in [1] to find solutions in the neighbourhood of the homoclinic one. The chaotic behaviour of this solution is proved and it is shown that a local analysis of this solution is sufficient to determine its position with respect to manifold Ω_0 containing the critical point. By linearization in the neighbourhood of the critical point, it is proved that the chaotic solution of perturbed system remains for all the time in the same halfspace! bounded by Ω_0. Our results can be easily applied also for discontinuous systems possessing homoclinic solution.

To our knowledge, position of a chaotic solution near homoclinic orbit has never been investigated and similar results has not yet been presented.

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On the spectral behavior of the Neumann Laplacian under mass density perturbation

Let Ω be a bounded open set in $\mathbb{R}^N$ of class C^2. We consider the classical eigenvalue problem

$$\begin{cases} -\Delta u = \lambda \rho u, & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = 0, & \text{on } \partial\Omega, \end{cases}$$

in the unknowns u (the eigenfunction) and λ (the eigenvalue). The parameter ρ is a positive bounded measurable function which can be understood as a mass density, in which case $M = \int_\Omega \rho dx$ represents the corresponding total mass. We discuss stability results for the dependence of the eigenvalues and eigenfunctions upon variation of ρ. In particular, we consider the case where M is fixed and $\rho = \rho_\varepsilon$, $\varepsilon > 0$ is a family of mass densities concentrating near the boundary of Ω as $\varepsilon \to 0$. We prove norm resolvent convergence of such Neumann problems to the appropriate limiting Steklov problem. In the case where Ω is a ball, explicit computations allow to prove differentiability of the eigenvalues upon variation of ε and provide formulas for the derivatives at $\varepsilon = 0$.

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The hidden symmetry analysis of Lax type integrable nonlinear dynamical systems within the Lie-algebraic, symplectic and differential-algebraic approaches

It is well known that hidden symmetry properties, related with symplectic, differential-geometric, differential-algebraic or analytical structures of nonlinear Hamiltonian dynamical systems on functional manifolds, such an infinite hierarchy of conservation laws and compatible Poissonian structures, often give rise to their Lax type integrability. This fact was extensively worked out by many researches during past half century and a very powerful so called inverse Lie-algebraic orbit method of constructing hierarchies of *a priori* Lax type integrable nonlinear dynamical systems was devised. Recently when studying integrability properties of infinite so called Riemann type hydrodynamical hierarchies, a new direct approach to testing the Lax type integrability of *a priori* given nonlinear dynamical system of special structure, based on treating the related both symplectic and differential-algebraic structures of involved differentiations, was recently suggested and devised by the author with his coworkers. By means of this technique the direct integrability problem was effectively enough reduced to the classical one of finding the corresponding compatible representations in suitably constructed differential rings. Concerning the mentioned above inverse Lie-algebraic orbit method, as its name says, consists in studying so called invariant orbits of the coadjoint group $\hat{G}$ action on a specially chosen element $l \in \mathcal{G}^*$, where $\mathcal{G}^*$ is the conjugate space to the Lie algebra $\mathcal{G}$ of a suitably chosen, in general formal, group $\hat{G}$. In other words, the main Lie-algebraic essence of this approach consists in considering functional invariance and related symplectic properties of these extended orbits in $\mathcal{G}^*$, generated by the given element $l \in \mathcal{G}^*$ and inherited from the standard Lie algebra structure of the set $\mathcal{G}$. From this point of view, subject to this *extension* scheme of constructing *a priori* Lax type integrable dynamical systems, it was natural enough to search for another way of constructing such systems, but based on a suitably chosen *reduction* construction of the corresponding coadjoint group $\hat{G}$ action on the already general element $l \in \mathcal{G}^*$. Happily, in the modern symplectic geometry such a *reduction* method was well developed many years ago by Marsden and Weinstein [1] and effectively applied to studying integrability properties of some nonlinear dynamical systems on finite-dimensional symplectic manifolds. Thus, a next step, consisting in developing this reduction method and applying it to the case of infinite dimensional dynamical systems on functional manifolds, was quite natural and effectively realized in [3]. The latter, in particular, made it possible to strongly generalize results of [2] and apply them to studying a new physically feasible and important model in modern quantum physics.

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Monogenic functions in a three-dimensional harmonic semi-simple algebra

Let $\mathbb{A}_1$ be a three-dimensional commutative associative Banach algebra over the field of complex numbers $\mathbb{C}$. Let $\{I_1, I_2, I_3\}$ be a basis of the algebra $\mathbb{A}_1$ with the multiplication table

$$I_k{}^2 = I_k,\ \ I_k I_j = 0,\ \ k, j = 1, 2, 3,\ \ k \neq j\,.$$

The unit of $\mathbb{A}_1$ is represented as the sum of idempotents: $1 = I_1 + I_2 + I_3$.

Algebra $\mathbb{A}_1$ is harmonic because there exist bases $\{e_1, e_2, e_3\}$ in $\mathbb{A}_1$ satisfying the condition $e_1{}^2 + e_2{}^2 + e_3{}^2 = 0$ and $e_k^2 \neq 0$, $n = 1, 2, 3$.

Let $E_3 := \{\zeta = xe_1 + ye_2 + ze_3 :\ x, y, z \in \mathbb{R}\}$ be a linear span in $\mathbb{A}_1$ over the field of real numbers $\mathbb{R}$. In what follows, $\zeta = xe_1 + ye_2 + ze_3$ and $x, y, z \in \mathbb{R}$.

Let Ω be a domain in E_3. We say that a continuous function $\Phi : \Omega \to \mathbb{A}_1$ is *monogenic* in Ω if Φ is differentiable in the sense of Gateaux in every point of Ω, i.e. if for every $\zeta \in \Omega$ there exists an element $\Phi'(\zeta) \in \mathbb{A}_1$ such that

$$\lim_{\varepsilon \to 0+0} \left(\Phi(\zeta + \varepsilon h) - \Phi(\zeta)\right)\varepsilon^{-1} = h\Phi'(\zeta) \quad \forall h \in E_3.$$

$\Phi'(\zeta)$ is the Gateaux derivative of the function Φ in the point ζ.

Consider three linear continuous multiplicative functionals $f_k : \mathbb{A}_1 \to \mathbb{C}$ for $k = 1, 2, 3$ such that $f_k(I_k) = 1, \quad f_k(I_j) = 0, \quad j = 1, 2, 3,\ k \neq j$.

We introduce the notation:

$$\xi_k := f_k(\zeta) = f_k(xe_1 + ye_2 + ze_3), \quad D_k := f_k(\Omega),$$

$$L_k : \{xe_1 + ye_2 + ze_3 : \mathrm{Re}\,\xi_k = 0\,, \mathrm{Im}\,\xi_k = 0\,,\ \ x\,, y\,, z \in \mathbb{R}\},\ \ k = 1, 2, 3.$$

Theorem 1. *In an arbitrary domain Ω convex in the direction of the straight line L_k for all $k \in \{1, 2, 3\}$, every monogenic function $\Phi(\zeta)$ can be explicitly constructed with using three holomorphic functions in the form:*

$$\Phi(\zeta) = F_1(\xi_1)I_1 + F_2(\xi_2)I_2 + F_3(\xi_3)I_3,$$

where $\xi_j = f_j(\zeta)$ and F_j is a function holomorphic in the domain D_j for $j = 1, 2, 3$

Theorem 2. *Let a domain $\Omega \subset E_3$ be convex in the direction of the straight line L_k for all $k \in \{1, 2, 3\}$ and a function $\Phi : \Omega \to \mathbb{A}_1$ be monogenic in Ω. Then Φ can be continued to a function monogenic in the domain $\Delta := \{\zeta = xe_1 + ye_2 + ze_3 :\ f_k(\zeta) \in D_k,\ k = 1, 2, 3\}$.*

Theorem 3. *For every monogenic function $\Phi : \Omega \to \mathbb{A}_1$ in an arbitrary domain Ω, the Gateaux n-th derivatives $\Phi^{(n)}$ are monogenic functions in Ω for any n.*

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Modelling - a problem not only for secondary school students

One of main objectives of mathematics education is achieving by students skills to solve problems encountered in everyday life. Finding a solution to the problem forces the creation of a mathematical model of a given situation. I decided to investigate whether students of mathematics, so those who have already gone through a sufficient stage of education needed to obtain the desired performance, possess such skills.

Respondents were asked to create a scheme that allows to calculate the cost of a school trip. The scheme was supposed to be so general and clear that it could be used by another organizer while planning a similar trip. Students, forced to create a mathematical model of a known everyday life situation, approached the problem in different ways. Therefore, the analysis of the work was carried out from different angles. Similarly as while solving the task, the respondents had to determine the data first - the constants and variables, and then to identify and precisely formulate the relationship between them so as to finally create the mathematical model that allows to obtain the desired result. I have carefully analysed those three stages of problem solving. Despite the diversity of solutions, common features can be found. The work of students shows that the majority of respondents are aware of the need to use variables to ensure the generality of the model, although specific data appear in some works. This indicates the difficulty of using generalizations. Despite the fact that more than a half of the works show a scheme that is based on a well thought out conception of solving the problem, which is properly presented with the use of variables, only four out of eleven schemes work correctly. Does it mean that students cannot mathematize? It is a problem that is certainly worth considering.

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Questions on Approximation by Reproducing Kernels of Clifford Hardy spaces

In the one complex variable setting there have existed well developed approximation results to functions in the Hardy spaces that are with the key words mono-components, adaptive Fourier decomposition, optimal rational approximation, Takenaka-Malmquist system, Beurling-Lax shift and backward shift invariant subspaces. Part of the results in one dimension can be generalized to multi-dimensional cases under the several complex variables and the Clifford algebra settings. But there are obstacles. The talk will recall old and new results and open questions in this research direction.

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Diffraction by periodic graphs

The talk is devoted the diffraction by graphs Γ imbedded in $\mathbb{R}^2$ periodic with respect to the action of the group $\mathbb{Z}^n, n = 1, 2$. We consider the Helmholtz equation

$$\mathcal{G}u(x) = \rho(x)\nabla \cdot \rho^{-1}(x)\nabla u(x) + a(x)u(x) = g(x), x \in \mathbb{R}^2 \backslash \Gamma \tag{1}$$

where ρ is a density of a medium, $a = \frac{\omega^2}{c^2}, \omega > 0$ is a frequency of harmonic vibrations, c is a velocity of the sound. We add to equation (1) transmission conditions on Γ

$$[u(t)]_{t \in \Gamma \backslash \mathcal{V}} = 0,$$

$$\left[\alpha(t)\frac{\partial u(t)}{\partial n_t}\right]_{t \in \Gamma \backslash \mathcal{V}} + \beta(t)u(t) = f(t), t \in \Gamma \backslash \mathcal{V}, f \in L^2(\Gamma)$$

where $\mathcal{V}$ is a set of vertices of the graph Γ, $\alpha \in PC(\mathbb{R}^2, \Gamma)$ the space of the bounded piece-wise continuous functions on $\mathbb{R}^2$ with discontinuities on $\Gamma, \beta \in PC(\Gamma, \mathcal{V})$ the space of bounded piece-wise continuous functions on Γ with discontinuities at vertices, $[\varphi(t)]_{t \in \Gamma \backslash \mathcal{V}}$ is the jump of φ at the point $t \in \Gamma \backslash \mathcal{V}$. We introduce single and double layer potentials connected with the operator $\mathcal{G}$ and reduce the transmission problem to a pseudo-differential equation on the graph Γ. We obtained necessary and sufficient conditions for the boundary pseudo-differential operators to be Fredholm in $L^2(\Gamma)$.

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Keywords: Helmholtz Operators, Periodic Graphs, Diffraction.

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Potential Type Operators in Weighted Variable exponent spaces on some Carleson Curves

We consider the operators of single and double-layer potentials acting in weighted variable exponent Lebesgue space $L^{p(\cdot)}(\Gamma, w)$ on some composed Carleson curves. We assume that the simple curves which form a node t_0 are slowly oscillating near the point t_0. We obtain the Fredholm criterion for operators $A = aI + D_{\Gamma,g} : L^{p(\cdot)}(\Gamma, w) \to L^{p(\cdot)}(\Gamma, w)$ where $D_{\Gamma,g}$ is the operator of the form

$$D_{\Gamma,g}u(t) = \frac{1}{\pi}\int_\Gamma \frac{g(t,\tau)\,(\nu(\tau), t-\tau)\,u(\tau)dl}{|t-\tau|^2}, t \in \Gamma,$$

where $\nu(\tau)$ is the inward unite normal vector to Γ at the point $\tau \in \Gamma \backslash \mathcal{F}$, dl is the oriented Lebesgue measure on Γ, $a : \Gamma \to \mathbb{C}, g : \Gamma \times \Gamma \to \mathbb{C}$ are a bounded functions with discontinuities at the nodes only.

We also consider the operators $A = aI + bL_\Gamma$ of the logarithmic potential where

$$L_\Gamma u(t) = \frac{1}{\pi}\int_\Gamma \log(|t-\tau|)u(\tau)dl, t \in \Gamma.$$

We wil give applications to the Dirichlet and Neumann problem with boundary function in $L^{p(\cdot)}(\Gamma, w)$ for domains with a simple connected boundaries having a finite set of oscillating singularities.

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Boundary value problems with state-dependent impulses

Impulsive differential equations have attracted lots of interest due to their important applications in many areas such as aircraft control, drug administration, and threshold theory in biology. A particular case of impulsive problems are *problems with impulses at fixed moments.* This occurs when the moments, at which impulses act in state variable, are known. Very different situation arises, when the impulses appear in evolutionary trajectories fulfilling a predetermined relation between state and time variables. This case, which is represented by state-dependent impulses, is discussed here. In particular, we investigate the solvability of *boundary value problems with state-dependent impulses.* As the methods used for problems with impulses acting at fixed points do not apply to problems with state-dependent impulses, only few paper dealing with state-dependent case may be found in the literature. Most of them consider periodic problems, see e.g. [1].

The main cause of difficulties in the investigation of problems with state-dependent impulses lies in the following fact: the operator, corresponding to the problem with state-dependent impulses which is constructed in a standard way (used for problems with fixed-time impulses), is not continuous. Therefore, in [2] and [3] we provide a new approach which makes possible to find sufficient conditions for solvability of the problem

$$z''(t) = f(t, z(t), z'(t)), \quad \text{for a.e. } t \in [a, b],$$
$$z(\tau_i+) - z(\tau_i) = J_i(\tau_i, z(\tau_i)), \quad z'(\tau_i+) - z'(\tau_i-) = M_i(\tau_i, z(\tau_i)), \quad \ell(z, z') = c,$$

where the points $\tau_1, \ldots, \tau_p$ depend on z through the equations

$$\tau_i = \gamma_i(z(\tau_i)), \quad i = 1, \ldots, p, \quad p \in N.$$

Here f fulfils the Carathéodory conditions, the impulse functions J_i, M_i, and the barriers γ_i, $i = 1, \ldots, p$, are continuous, $c \in R^2$, and ℓ is a linear and bounded operator.

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About study's problems of stochastic line in Russian schools and universities

A long-term work with pupils, teachers and students testifies that all of them collide with serious problems in learning stochastic notions and methods. In our opinion these problems are conditioned by the following circumstances:

1. As we know child's mentality that works before forming of conceptual apparatus bears probabilistic nature. Due to this, child's stochastic view about real world should be developed continuously: kindergarten - elementary school - secondary school - university. In elementary school genetic base of stochastic notions must be opened in the process of children's interac-

tion with real sets and values by calculation of element's number in subsets, its seriations and classifications.

2. In secondary school content of study material must be developed in the following logical line: Theory of Combinations - Statistics - Theory of Probability. In our point of view, pupil's combinatorial and statistic ideas must become the base of learning probabilistic models as Statistics deals with real data of real process of random or nonrandom effects, and its mathematical models are learnt by Theory of Probability.

3. Now there is not functioning books in Russian schools in which the stochastic line would be realized persistently: kindergarten - elementary school - secondary school - university.

4. Because of this students have big troubles, especially psychological problems, in studies of Mathematical Statistics' and Theory of Probability's courses. And it is naturally because they get used to quick knowledge's formalization in learning Maths at schools. This formalization is caused by strict logic of study material.

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Transvector algebras in Clifford analysis

In higher spin Clifford analysis, it is known that any representation of the Spin(m)-group with a half-integer highest weight $\lambda' = \left(l_1 + \frac{1}{2}, \ldots, l_k + \frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2}\right)$ can be modelled by the space of simplicial monogenic polynomials in k vector variables $u_1, \ldots, u_k$, homogenic of degree l_i in u_i for each $i \in \{1, \ldots, k\}$. We denote this space by $\mathcal{S}_\lambda$, see [1].

The theory of generalised gradients (e.g. [2, 4]) tells us that the only conformally invariant first order differential operators acting on the representation $\mathbb{R}^m \otimes \mathcal{S}_\lambda$ (which can be identified with the space of polynomials $\mathcal{C}^\infty(\mathbb{R}^m, \mathcal{S}_\lambda)$) are the higher spin Dirac operator $\mathcal{Q}_\lambda$, at most k twistor operators, and at most $k+1$ dual twistor operators.

In this talk, it will be shown that these differential operators can be seen as generators of a suitable algebra, hereby generalising the well-known fact that the classical Dirac operator and its symbol generate the orthosymplectic Lie algebra $\mathfrak{osp}(1,2)$. To do so, we will use the extremal projection operator and its relation to transvector algebras (see e.g. [3, 5, 6]).

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Riesz type potential operators in generalized grand Morrey spaces

In this talk we introduce generalized grand Morrey spaces in the framework of quasimetric measure spaces, in the spirit of the so-called grand Lebesgue spaces introduced in [1]. Boundedness of Riesz type potential operators are obtained in the framework of homogeneous and also in the nonhomogeneous case in the generalized grand Morrey spaces.

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About one class of complex two-dimensional integral equation with fixed boundary singular kernels

Let D denote the disc $D = \{|z| < R\}$ and $\Gamma = \{|z| = R\}$ its boundary. In D, we consider the following two-dimensional complex integral equation

$$\varphi(z) + \frac{1}{2\pi}\iint_D \left[p + q\ln\left(\frac{R-r}{R-\rho}\right)\right] \frac{exp[i\psi]\overline{\varphi(t)}}{(R-\rho)(t-z)} d\xi d\eta = f(z), \qquad (1)$$

where p, q – is real parameters, $\psi = argt$, $t = \xi + i\eta$, $z = x + iy$, $r^2 = x^2 + y^2$, $\rho^2 = \xi^2 + \eta^2$, $f(z)$ are given function, $\varphi(z)$ unknown function.

In this work the solution integral equation (1) in depend from signs parameters and the roots of the corresponding characteristic equation found in explicit form. Investigation behavior obtained solution in neighborhood singular boundary. In the case, when general solution equation (1) depend from arbitrary constants, for equation (1) stend and investigation different boundary value problems. In the case, when in (1) $f(z) = f(r)$ and expend to uniformly convergent generalized power series by power $(R - r)$, then for integral equation (1) found the solution, which representable in generalized power series by power $(R - r)$.

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About one class two dimensional Voltera type Integral Equation with boundary singular lines

Let D denote rectangle $D = \{a < x < a_0,\ b < y < b_0\}$, $\Gamma_1 = \{a < x < a_0,\ y = b\}$, $\Gamma_2 = \{x = a,\ b < y < b_0, \}$. In D we conside integral equation

$$u(x,y) + A_1 \int_a^x \frac{u(t,y)}{t-a} dt + A_2 \int_a^x \ln\left(\frac{x-a}{t-a}\right) \frac{u(t,y)}{t-a} dt + B_1 \int_b^y \frac{u(x,s)}{s-b} ds +$$
$$+ B_2 \int_b^y \ln\left(\frac{y-b}{s-b}\right) \frac{u(x,s)}{s-b} ds + C_1 \int_a^x \frac{dt}{t-a} \int_b^y \frac{u(t,s)}{s-b} ds + C_2 \int_a^x \frac{dt}{t-a} \int_b^y \ln\left(\frac{y-b}{s-b}\right) \frac{u(t,s)}{s-b} ds +$$
$$+ C_3 \int_a^x \ln\left(\frac{x-a}{t-a}\right) \frac{dt}{t-a} \int_b^y \frac{u(t,s)}{s-b} ds + C_4 \int_a^x \ln\left(\frac{x-a}{t-a}\right) \frac{dt}{t-a} \int_b^y \ln\left(\frac{y-b}{s-b}\right) \frac{u(t,s)}{s-b} ds = f(x,y), \quad (1)$$

where A_i, B_i, C_j, $i = 1,2$, $j = \overline{1,4}$ – are given constants, $f(x,y)$ is a given function.

Solution Integral Equation (1) will be seek in class function $u(x,y) \in C(\overline{D})$, correspondingly vanishing in Γ_1 and Γ_2.

In the case $A_2 = B_2 = C_2 = C_3 = C_4 = 0$ Integral Equation (1) is a model two dimensional Voltera type Integral Equation, which has been studied yet.

In the case, when parameters in the integral equation connected between himself certain form, in depend from value signs numbers this parameters and roots charactestic equations, general solution of the homogeneous equation contain a few arbitrary functions one variable and for some values has a unigue solution.

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On the Schroedinger equation with one-dimensional potential.

A new method to resolve the stationary Schroedinger equation converted to the nonlinear Riccati equation has been described. The solution has been obtained by means of a new kind transformation which preserve the form of the Riccati equation. Some new solvable potentials for the Schroedinger and Klein-Gordon equations have also been discussed.

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Fractional derivatives of Colombeau generalized stochastic processes defined on $\mathbb{R}^+$

We consider Caputo and Riemann-Liouville fractional derivatives of a Colombeau generalized stochastic process G defined on $\mathbb{R}^+$. We give proper definitions and prove that both are Colombeau generalized stochastic processes themselves.

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L-weighted Hölder spaces

We introduce and study properties of a new class of weighted Hölder spaces, where the weights are regularly varying functions ([1]). The analysis is performed by the wavelet transform and generalized Littlewood-Paley pairs. As main tool we use pointwise weak regularity properties of a vector-valued distributions and their tauberian characterizations in terms of the wavelet transform ([2]).

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Hadamard well-posedness for a hyperbolic equation of viscoelasticity with supercritical sources and damping

Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with boundary Γ of class C^2. We consider the following model of viscoelasticity:

$$\begin{cases} u_{tt} - k(0)\Delta u - \int_0^\infty k'(s)\Delta u(x,t-s)ds + g(u_t) = f(u), \quad \text{in } \Omega \times (0,\infty), \\ u(x,t) = 0, \quad \text{on } \Gamma \times \mathbb{R}, \\ u(x,t) = u_0(x,t), \quad \text{in } \Omega \times (-\infty, 0], \end{cases} \tag{1}$$

where u denotes a scalar component of the elastic deformation vector, g is a monotone feedback, and $f(u)$ is a source. The relaxation function $k(s)$ satisfies the typical conditions: $k(0), k(\infty) > 0$ and $k'(s) \leq 0$ for all $s > 0$. The memory term $\int_0^\infty k'(s)\Delta u(x,t-s)ds$ quantifies the viscous resistance and provides a weak form of energy dissipation. It also emphasizes the full past history as time goes to $-\infty$, as opposed to the finite-memory model where the history is taken only over the interval $[0,t]$.

We employ the theory of monotone operators and nonlinear semigroups, combined with energy methods to establish the existence of a unique local weak solution. In addition, it is shown that the solution depends continuously on the initial data, and is global provided the damping dominates the source in appropriate sense.

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On approximation of YAKUBOVICH *transforms*

New modification of KONTOROVICH–LEBEDEV and LEBEDEV–SKALSKAYA integral transforms was introduced by S.B.Yakubovich [1]. These transforms contain modified BESSEL functions $K_{\frac{1}{4}+i\tau}(x)$ and $K_{\frac{3}{4}+i\tau}(x)$ and their real and imaginary parts as a kernels. The vector tau method approach is used for the approximation and calculation of these functions. This approach is based on the general tau method's computational scheme and canonical vector-polynomial notion [2,3]. We obtain the system of two differential equations and then the system of two VOLTERRA integral equations for the determination of the polynomial approximation of the kernels. These results may be used for the application of YAKUBOVICH transforms to the solution of boundary value problems of mathematical physics. This work was supported by a Thematic Programme on Inverse Problems and Imaging of FIELDS Institute.

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Some ophthalmic applications of modified BESSEL *functions*

Mathematical functions may be used for the simplification of some ophthalmological biomechanical models. So modified Bessel functions $K_0(z)$ and $I_0(z)$ are used for the simulation of intraocular pressure tonometry measurement by MACKLAKOV method [1]. The special functions, orthogonal polynomials and integral transforms are useful for the analytical and numerical solution of some boundary value problems in wedge domains which may have an ophthalmic interpretation. The morphometry of transplant endothelium after keratoplasty is analysed [2]. Internet and mobile possibilities for the treatment of ophthalmic diseases are described [3]. This work was supported by a Thematic Programme on Inverse Problems and Imaging of FIELDS Institute.

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Hopf bifurcation for dissipative first-order hyperbolic systems

We consider boundary value problems for semilinear first-order hyperbolic systems of the type

$$\begin{aligned} &\partial_t u_j + a_j(x,\lambda)\partial_x u_j + b_j(x,\lambda,u) = 0,\ x \in (0,1),\ j = 1,\dots,n,\\ &u_j(0,t) = \sum_{k=m+1}^{n} r_{jk} u_k(0,t), \quad j = 1,\dots,m,\\ &u_j(1,t) = \sum_{k=1}^{m} r_{jk} u_k(1,t), \quad j = m+1,\dots,n \end{aligned}$$

with smooth coefficient functions a_j and b_j such that $b_j(x,\lambda,0) = 0$. We state conditions for Hopf bifurcation, i.e. for existence, local uniqueness (up to phase shifts), smoothness and smooth dependence on λ of time-periodic solutions bifurcating from the zero stationary solution. Furthermore, we derive a formula which determines the bifurcation direction, and we describe applications to semiconductor laser dynamics.

The proof is done by means of a Lyapunov-Schmidt reduction procedure. For this purpose, Fredholm properties of the linearized system and implicit function theorem techniques are used.

There are several distinguishing features of the proofs of Hopf bifurcation theorems for hyperbolic PDEs in comparison with those for parabolic PDEs or for ODEs: First, the question of Fredholm solvability of the linearized problem (in appropriate spaces of time-periodic functions) is essentially more difficult. Second, the question if a non-degenerate time-periodic solution of the nonlinear problem depends smoothly on the system parameters is much more delicate. And third, a sufficient amount of dissipativity is needed in order to prevent small denominators from coming up, and we present an explicit sufficient condition for that in terms of the data of the PDEs and of the boundary conditions.

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Well-posedness for degenerate Schrödinger equations

We are interested in the initial value problem for Schrödinger type equations

$$\frac{1}{i}\partial_t u - a(t)\Delta_x u + \sum_{j=1}^{n} b_j(t,x)\partial_{x_j} u = 0$$

with $a(t)$ vanishing of finite order at $t = 0$ proving the well-posedness in Sobolev and Gevrey spaces according to the behavior of the real parts $\Re b_j(t,x)$ as $t \to 0$ and $|x| \to \infty$. To get the results we determine a suitable weight function depending on different zones of the extended phase space, we form after a change of variables the conjugation by pseudo-differential operators of infinite order and apply sharp Gårding inequality to prove L^2 well-posedness of an auxiliary Cauchy problem.

Moreover, we discuss the application of our approach to the case of a general degeneracy.

[1] M. Cicognani, M. Reissig: *Well-posedness for degenerate Schrödinger equations*, 19pp., submitted.

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Semi-linear damped waves

We study the Cauchy problem for the semi-linear damped wave equation with source term

$$u_{tt} - \triangle u + b(t)(-\Delta)^\sigma u_t = f(u), \quad u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x),$$

with $\sigma \in [0,1]$ in space dimension $n \geq 2$. We are interested in the influence of σ on the critical exponent p_{crit} in $|f(u)| \approx |u|^p$. This critical exponent is the threshold between global existence in time of small data solutions and blow-up behavior for some suitable range of p. First we discuss the case $\sigma = 0$ with an effective dissipation $b(t)u_t$ (see [1]). Then we devote to structurally damped wave models ($\sigma \in (0,1)$ and $b(t) \equiv \mu > 0$). In particular, we propose new strategies concerning a suitable choice of the energies (see [2]). Optimality of our results are given in special cases by applying test function method. Some new proposals for research topics complete the lecture.

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On the quaternionic $F(p,q,s)$ spaces

Let $\mathbb{B}$ the unit ball in $\mathbb{R}^3$ and identify each vector $\mathbf{x} = (x_0, x_1, x_2) \in \mathbb{R}^3$ with the reduced quaternion $\mathbf{x} = x_0 + x_1\mathbf{i} + x_2\mathbf{j}$. Let ∂ and $\overline{\partial}$ the reduced Cauchy-Riemann operators and for $\mathbf{a} \in \mathbb{B}$ define the Möbius transformation, $\phi_{\mathbf{a}} : \mathbb{B} \to \mathbb{B}$, by

$$\phi_{\mathbf{a}}(\mathbf{x}) = (\mathbf{a} - \mathbf{x})(1 - \bar{\mathbf{a}}\mathbf{x})^{-1} .$$

Define

$$G(\mathbf{x}, \mathbf{a}) = \frac{1}{4\pi}\left(\frac{1}{|\phi_{\mathbf{a}}(\mathbf{x})|} - 1\right)$$

the modified fundamental solution of the Laplacian in $\mathbb{R}^3$.

Let $f : \mathbb{B} \to \mathbb{H}$ be a hyperholomorphic function, that is $\partial f = 0$ and for $2 \leq p < \infty$, $-2 < q < \infty$, $0 < s < 3$ define

$$I_{p,q,s}(f, \mathbf{a}) = \int_{\mathbb{B}} |\overline{\partial} f(\mathbf{x})|^p (1 - |\mathbf{x}|^2)^q G^s(\mathbf{x}, \mathbf{a})\, dv(\mathbf{x}) .$$

We say that f belongs to the space $F_g(p,q,s)$ if

$$\sup_{\mathbf{a}\in\mathbb{B}} I_{p,q,s}(f,\mathbf{a}) < \infty \ .$$

In this talk we present some properties and characterizations of these spaces, that generalizes the Zhao-type spaces [3]. The particular quaternionic cases $Q_s = F(2,0,s)$ and $F(p, \frac{3p}{2} - q, q)$ were studied in [2] and [1].

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Multi-Dimensional Stockwell Transforms with Applications to Instantaneous Frequencies of Muti-Dimensional Signals

Stockwell transforms, as hybrids of Gabor transforms and wavelet transforms introduced in 1994, have been used extensively in geophysics and medical imaging. There are many results regarding the one-dimensional Stockwell transforms both from a theoretical and an applicative perspective. In this talk we introduce multi-dimensional Stockwell transforms that include the one-dimensional ones and the multi-dimensional Gabor transforms as special cases. We are mainly interested in providing a somehow "natural" setting in which to study the Stockwell transforms. Continuous inversion formulas for the multi-dimensional Stockwell transforms under different set of hypothesis as well as some crucial examples will be provided. Furthermore we will enlight a connection between Stockwell transforms and instantaneous frequency of a signal.

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■**M. Manuela Rodrigues** - Center for Research and Development in Mathematics and Applications, Department of Mathematics, University of Aveiro - Portugal, email: `mrodrigues@ua.pt`

Study of some properties related to the Whittaker integral transform

We introduce Weierstrass transform associated with the Whittaker integral transform and we examine some of its properties. We established the Mellin transform-Parseval's formula for the Whittaker integral transform.
(Joint work with L.P. Castro and S. Saitoh)

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Higher spin generalisations of the Laplace operator

Clifford analysis has become a branch of multi-dimensional analysis in which far-reaching generalisations of the classical Cauchy-Riemann operator in complex analysis are studied from a function theoretical point of view. Without claiming completeness, one could say that the theory focuses on first-order conformally invariant operators acting on functions taking values in irreducible representations for the spin group. This then leads to function theories refining (poly-)harmonic analysis on $\mathbb{R}^m$. In our talk we will explain how to extend these results to a certain type of second-order conformally invariant operators, leading to analogues of the Rarita-Schwinger function theory for functions taking values in the space of harmonics (at the same time generalising harmonic analysis for Δ_m), see e.g. [2, 3].

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Complex-analytic model for Hele-Shaw moving boundary value problem

It will be reported a survey of the recent results on the complex-variable Hele-Shaw moving boundary value problem (see [1], [3]). Main attention will be paid to the method of reduction to the abstract Cauchy-Kovalevsky problem and to study of this problem in an appropriate scale of Banach spaces [2]. In particular, this approach is applied in the case of the flow in the channel with non-smooth boundary and in presence of obstacles in the flow.

Acknowledgement. The work is partially supported by the FP7-PEOPLE- 2009-IAPP grant PIAP-GA-2009-251475 HYDROFRAC.

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■ **Edixon Rojas** Pontificia Universidad Javeriana, Bogotá, Colombia, email: `edixon.rojas@javeriana.edu.co`

On the stability of a nonlinear Volterra integral equation

In this talk we are going to analyze the Hyers-Ulam-Rassias types of stability for nonlinear, nonhomogeneous Volterra integral equations with delay on finite intervals. In order to attain our goals, we are going to use a fixed point iterative process.

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■ **Grigori Rozenblum** Gothenburg, Sweden, email: Grigori@chalmers.se

Finite rank Toeplitz operators in the Fock space

Since 2008 it is known that any finite rank Toeplitz operator in the Bergman space of analytic function in the disk or polydisk must zero symbol. We consider finite rank operators with distributional symbol in the Fock space. It is shown that the symbol of such operator is a finite combination of delta distributions and its derivatives. Some interesting problems in complex analysis arise. The complete formulations and proofs are contained in [1]-[3].

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Matrix representation as a sampling for operators on some function spaces

For many operators acting on reproducing kernel Hilbert spaces their traditional matrix representation in orthonormal bases seems less useful than the representation using frames. Further gains in simplicity are achieved when one uses frames corresponding to normalized reproducing kernels at some lattices of points. The latter are relatively well understood due to atomic decompositions constructed in the majority of studied function spaces.

Here matricial representation of operators in these frames provides a natural link between the analysis based on the Berezin transform and its discretised version. To support this claim some results on the membership in Schatten-von Neumann classes will be presented.

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Nikolskii inequality and Besov spaces on compact homogeneous manifolds

In this talk we will establish Nikolskii inequality on compact Lie groups and on compact homogeneous manifolds, and apply it to analyse embedding properties between Besov, Wiener and Beurling spaces. This is joint work with Sergey Tikhonov and Erlan Nursultanov.

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On an application of quality of service characteristics of stochastic networks with dependent service times

Broadband wireless data transmission network for providing of automobile transport system safety is considered. The network operate under IEEE802.11n-2012 protocol that guarantees high-speed transmission of multimedia information from automatic stationary and mobile systems of traffic control. A model of open stochastic network with dependent service times, proposed in [1] for the network quality of service evaluation is used. Based on product form representation of the steady state probabilities of such system, proved in [1, 2], the formulas and algorithms for the main system characteristics such as: mean marginal queue length and mean sojourn times in the system nodes, as well as mean packages delivery time in the whole network are proposed. Results of some numerical experiments will be shown.

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■ **Natalia Rylko** - Pedagogical University of Cracow, email: `nrylko@gmail.com`

Piezoelectric fibrous composites

Anti-plane shear of piezoelectric fibrous composites is theoretically investigated. Geometry of composites is described by two–dimensional geometry in a section perpendicular to the unidirectional fibres. First, the piezoelectric problem is written in the form of the vector–matrix $\mathbb{R}$–linear problem in a class of double periodic functions. In particular, application of the zero–th order solution to the $\mathbb{R}$–linear problem yields a vector–matrix extension of the famous Clausius–Mossotti approximation. The vector–matrix problem is decomposed onto two scalar $\mathbb{R}$–linear problem. This reduction allows us to directly apply all the known exact and approximate analytical results for scalar problems to establish high order formulae for the effective piezoelectric constants. The special attention is paid to non–overlapping disks embedded in a two–dimensional background.

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Toward an extension of Schur analysis to the quaternionic setting

Schur analysis can nowadays be seen as a collection of results pertaining to Schur functions treated in various setting e.g. one and several complex variables, compact Riemann surfaces just to name a few. It has also been treated in the quaternionic case, by means of the so-called

Fueter regular functions. Recently, we started the study of Schur functions in the framework of the so-called slice hyperholomorphic (or slice regular) functions of a quaternionic variable, see [1], [2], [3], [4]. Using this class of functions we can introduce reproducing kernel Hilbert spaces, define Schur multipliers and describe their realizations. Indeed, slice hyperholomorphic functions allow to write realizations in terms of a suitable resolvent, the so called S-resolvent operator and to extend several results from the complex case to the quaternionic case that we shall discuss in the talk.

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The globally hyperbolic splitting in non-smooth Lorentzian manifolds

Classically (in smooth Lorentzian geometry), a Lorentzian manifold (M, g) is globally hyperbolic iff it splits as $\mathbb{R} \times S$, where S is a spacelike Cauchy hypersurface and thus (M, g) is isometric to $(\mathbb{R} \times S, -\beta dt^2 + h_t)$. Here $\beta : \mathbb{R} \times S \to \mathbb{R}$ is a smooth positive function, $t : \mathbb{R} \times S$ is the projection on the first factor and h_t is a t-dependent Riemannian metric on every level set $S_t = t \times S$ (which is also a Cauchy hypersurface). This notion of global hyperbolicity is well-suited for proving (unique) solvability of the Cauchy problem. Therefore it was recently generalized to non-smooth Lorentzian manifolds in the Colombeau setting. In this talk we will discuss this generalized hyperbolic metric splitting and its appropriateness when dealing with concrete non-smooth spacetimes.

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Some oscillation results on first order differential equations with deviating arguments

Some oscillation results on first order delay differential equations are considered. Similar results are obtained for differential equations of first order both containing delay and advance term.

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Colombeau algebras of generalized functions based on Gelfand-Shilov spaces

The aim of this paper is to introduce and study algebras of generalized functions based on the spaces of Gelfand-Shilov $\mathcal{S}^{\sigma}$, we also give some characterizations of these algebras. This work is the continuation of the paper [3].

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■ **Saburou Saitoh** University of Aveiro, email: saburou.saitoh@gmail.com

Reproducing kernels and discretization

Based on our recent results, we will introduce a new discretization method using the theory of reproducing kernels with concrete applications and numerical experiments by Professor Fujiwara. In particular, we shall refer to a method how to catch smoothness properties and analyticity of functions by computers, and a new type sampling theory.

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On the inclusions of some Lorentz mixed normed spaces and Wiener-Ditkin sets

In this paper, we consider sufficient and necessary condition for the inclusion between Lorentz spaces with mixed norms for two different product measures on $(X \times Y, \mathcal{A} \times \mathcal{B})$. Later, we discuss the Wiener-Ditkin sets of Lorentz spaces with mixed norms and Lorentz spaces.

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A orthonormal basis for Radial Basis Function approximation

It is well known that Radial Basis Function interpolants suffer of bad conditioning if the simple basis of translates is used. The recent work [2] gives a quite general way to build stable, orthonormal bases for a subset $\mathcal{N}_K(X)$, X a finite set in Ω, of the native Hilbert space $\mathcal{N}_K(\Omega)$, based on a factorization of the kernel matrix A_K. Starting from that setting we described in [1] a particular $\mathcal{N}_K(\Omega)$-orthonormal, $\ell_2^w(X)$-orthogonal basis that arises from a weighted singular value decomposition of A_K. This basis is related to a discretization of the compact operator $T_K : \mathcal{N}_K(\Omega) \to \mathcal{N}_K(\Omega)$,

$$T_K[f](x) = \int_\Omega K(x,y)f(y)dy \quad \forall x \in \Omega$$

and provides a connection with the continuous basis that arises from an eigen-decomposition of T_K.
Furthermore, this basis allows the extraction of a suitable sub-basis which preserves the approximation capability of the full one, while giving better results in terms of stability of the approximant.

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Ideals in the ring of generalized constants with continuous parametrization

In this paper we continue investigation of the maximal and prime ideals in the ring of generalized constants with.continuous coeficients the main result is to prove that the closure of any prime ideal is a maximal one and present a correspondence between some filters of closed subsets of (0,1) and ideals as well as the corespondance of closures for sharp topology and the closure in the set of subsets of (0,1) for a topology we introduce the so called extension topology.
This abstract concerns a common work by A.Khelif, H.Vernaeve and D. Scarpalezos.

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Hitchin fibration and real forms through spectral data.

The talk will be dedicated to the study of the moduli space of G-Higgs bundles and the Hitchin fibration through spectral data. We shall begin by introducing Higgs bundles following [Hit87], and defining the spectral data associated to G_c-Higgs bundles, for G_c a complex semisimple Lie group as in [Hit87a, Hit07]. Then we shall define G-Higgs bundles, for G a real form of a complex Lie group, recall general properties, and introduce their associated spectral data as given in [Sc13]. Through some examples we shall see applications of this new geometric way of understanding the moduli space using spectral data. In particular, we will consider G-Higgs bundles for G a split real form, for $G = SL(2, \mathbb{R})$ (following [Sc11]), and for $G = U(p, p), Sp(2p, 2p)$ (following [Sc13, Sc13a, Sc13b]). Time permitting, we will mention some further applications of the spectral data approach in relation to Langlands duality following [BaSc13].

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[Sc13b] L.P. Schaposnik, *Spectral data for Sp(2p,2p)-Higgs bundles*, *preprint.*

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Boundary value problems for the Cimmino system via quaternionic analysis

Cimmino system originates with the work of Italian mathematician G. Cimmino from 1941 (cf [1]). Later on, the motivation of the subject came from Dragomir and Lanconelli [2]. We study Cimmino system in the quaternionic setting. We discuss on Cauchy type integrals, boundary value problems related to the Cimmino system of partial differential equations.

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The Space of Initial Data for Parabolic Differential-Difference Equations

Let Q be a bounded Lipschitz domain in $\mathbb{R}^n$, $n \in \mathbb{N}$. We consider the problem

$$\frac{du}{dt} + A_R u = f, \qquad u(0) = \varphi, \tag{1}$$

where A_R is a differential-difference operator (see [1]) with Robin boundary condition, $f \in L_2(Q \times (0,T))$, $\varphi \in L_2(Q)$, and $0 < T < \infty$.

Let us remind that a variational solution u to problem (1) is said to be a strong solution if $u \in D(A_R)$ for almost all $t \in (0,T)$ and $u_t \in L_2(Q \times (0,T))$. We prove that if the operator A_R is strongly elliptic, then a strong solution to problem (1) exists if and only if $\varphi \in H^1(Q)$. In order to prove this statement, it is sufficient to show that $[L_2(Q), D(A_R)]_{1/2} = H^1(Q)$. This equality is equivalent to the Kato conjecture on the square root of the operator (see [2]). We note that smoothness of solutions of the equation $A_R v = F$ can be violated inside of the domain Q. Moreover, in case of incommensurable shifts, it can be violated on an almost everywhere dense set in Q. Thus, as in the case of strong elliptic differential operators with bounded measurable coefficients a domain $D(A_R)$ can not be represented in an explicit form.

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Computability of Solution Operators of Boundary-value Problems for Symmetric Hyperbolic Systems of PDEs

We prove computability of the solution operators of symmetric hyperbolic systems [1] with dissipative boundary conditions in a cube, in the exact sense of computable analysis [2].

Theorem. *Let $M_\varphi > 0$, $p \geq 2$ be integers, $Q = [0,1]^m$ be the unitary cube in $\mathbb{R}^m$. For $i = 1, 2, \ldots, m$ let A, B_i be fixed real computable symmetric matrices with $A > 0$, and $\Phi_i^{(1)}$ (respectively, $\Phi_i^{(2)}$) be fixed rectangular real computable matrices with the number of rows equal to the number of positive (respectively, negative) eigenvalues of the matrices $A^{-1}B_i$. Then the operator $\varphi \mapsto \mathbf{u}$ sending any function $\varphi \in C^{p+1}(Q)$ such that $||\frac{\partial \varphi}{\partial x_i}||_s \leq M_\varphi$, $||\frac{\partial^2 \varphi}{\partial x_i \partial x_j}||_s \leq M_\varphi$, $i, j = 1, 2, \ldots, m$, to the unique solution $\mathbf{u} \in C^p(H, \mathbb{R}^n)$ of the boundary-value problem with dissipative boundary conditions*

$$\begin{cases} A\frac{\partial \mathbf{u}}{\partial t} + \sum\limits_{i=1}^{m} B_i \frac{\partial \mathbf{u}}{\partial x_i} = f, \\ \mathbf{u}|_{t=0} = \varphi(x_1, \ldots, x_m), \\ \Phi_i^{(1)} \mathbf{u}(x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_m, t) = 0, \\ \Phi_i^{(2)} \mathbf{u}(x_1, \ldots, x_{i-1}, 1, x_{i+1}, \ldots, x_m, t) = 0, \\ i = 1, 2, \ldots, m, \end{cases}$$

is a computable partial function from the space $C_s(Q, \mathbb{R}^n)$ to $C_{sL_2}(H, \mathbb{R}^n)$.

Here $||\cdot||_s$ is the sup-norm, $||\cdot||_{sL_2}$ is the sup-norm on t and L_2-norm on Q. Dissipativity of the boundary conditions means that $(B_i \mathbf{u}, \mathbf{u}) \leq 0$ for $x_i = 0$ and $(B_i \mathbf{u}, \mathbf{u}) \geq 0$ for $x_i = 1$, for all $i = 1, 2, \ldots, m$.

The proof is based on facts of numeric analysis and of the theory of PDEs [1, 3], and on the strong constructivizability of the field of algebraic real numbers. Since dissipative boundary-value problems for the wave equation can be reduced to dissipative boundary-value problems of symmetric hyperbolic systems, our result provides an answer to an open question of computable analysis posed by Weihrauch and Zhong in 2002.

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Local properties of some class of mappings with non-bounded characteristics of quasiconformality

Let D be a domain in $\mathbb{R}^n$, $n \geq 2$, and $f : D \to \mathbb{R}^n$ be a continuous mapping. Recall a mapping $f : D \to \mathbb{R}^n$ is said to be *local homeomorphism* in D, if for every $x_0 \in D$ there is $\delta > 0$ such that $f|_{B(x_0,\delta)}$ is homeomorphic; here $B(x_0, \delta)$ denotes the ball in $\mathbb{R}^n$ centered at x_0 with radius δ. Denote by $S(x_0, r_1)$ and $S(x_0, r_2)$ the corresponding boundaries of the ring $A(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\}$ and let Γ be a family of paths γ in $\mathbb{R}^n$ which join $S(x_0, r_1)$ and $S(x_0, r_2)$ in $A(x_0, r_1, r_2)$. Given a (Lebesgue) measurable function $Q : D \to [0, \infty]$, a mapping $f : D \to \mathbb{R}^n$ is called *ring Q–mapping at a point* $x_0 \in D$ if the conformal modulus satisfies the following inequality

$$M\left(f(\Gamma(S(x_0, r_1), S(x_0, r_2), A(x_0, r_1, r_2)))\right) \leq \int\limits_{A(x_0,r_1,r_2)} Q(x) \cdot \eta^n(|x - x_0|)\, dm(x)$$

for any $A(x_0, r_1, r_2)$, $0 < r_1 < r_2 < r_0 = \operatorname{dist}(x_0, \partial D)$, and for every Lebesgue measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that $\int\limits_{r_1}^{r_2} \eta(r)dr \geq 1$. We also denote by $q_{x_0}(r)$ the integral average of $Q(x)$ over $|x - x_0| = r$, i.e. $q_{x_0}(r) := \frac{1}{\omega_{n-1} r^{n-1}} \int\limits_{|x-x_0|=r} Q(x)\, dS$, where ω_{n-1} is the area of $\mathbb{B}^n = B(0, 1)$. In what follows, we use in $\overline{\mathbb{R}^n} = \mathbb{R}^n \bigcup\{\infty\}$ the *spherical (chordal) metric* $h(x, y) = |\pi(x)-\pi(y)|$ where π is the stereographic projection of $\overline{\mathbb{R}^n}$ onto the sphere $S^n(\frac{1}{2}e_{n+1}, \frac{1}{2})$ in $\mathbb{R}^{n+1}$. The *spherical (chordal) diameter* of a set $E \subset \overline{\mathbb{R}^n}$ is $h(E) = \sup_{x,y\in E} h(x, y)$.

Let (X, d) and (X', d') be metric spaces with distances d and d', respectively. A family $\mathfrak{F}$ of mappings $f : X \to X'$ is said to be *equicontinuous at a point* $x_0 \in X$ if for every $\varepsilon > 0$ there is $\delta > 0$ such that $d'(f(x), f(x_0)) < \varepsilon$ for all $f \in \mathfrak{F}$ and $x \in X$ with $d(x, x_0) < \delta$. In what follows we consider that $X = D$, $X' = \overline{\mathbb{R}^n}$, $d(x, y) = |x - y|$ be Euclidean metric and $d'(x, y) = h(x, y)$ be chordal metric.

Denote by $\mathfrak{F}_{Q,\Delta}(x_0)$ a family of all local ring Q-homeomorphisms $f : D \to \overline{\mathbb{R}^n}$ at x_0 with $h(\overline{\mathbb{R}^n} \setminus f(D)) \geq \Delta > 0$. The main result of the talk is the following

Theorem. *A family $\mathfrak{F}_{Q,\Delta}(x_0)$ is equicontinuous at $x_0 \in D$ whenever at least one of the following conditions holds: 1)* $\int\limits_0^{\varepsilon(x_0)} \frac{dt}{t q_{x_0}^{1/(n-1)}(t)} = \infty$ *for some* $\varepsilon(x_0) > 0$, $\varepsilon(x_0) < \operatorname{dist}(x_0, \partial D)$; *2)* $q_{x_0}(r) \leq C \cdot \log^{n-1} \frac{1}{r}$ *as* $r \to 0$ *for some* $C > 0$; *3)* $Q \in BMO(D)$.

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The Riemann–Hilbert boundary value problem with singularities of coefficients and index.

The Hilbert boundary value problem for the half-plane consists in evaluation of all analytical in upper half-plane D functions $F(z)$ satisfying relation

$$a(t)ReF(t) - b(t)ImF(t) = c(t), \quad t \in L, \tag{1}$$

where L stands for the real axis, the functions $a(t)$, $b(t)$, $c(t)$ are defined for $t \in L$, and $a^2(t) + b^2(t) \neq 0$. The solution of problem (1) is known for the following cases: (a) the functions $a(t)$, $b(t)$, $c(t)$ belong to the Hölder class $H_L(\mu)$, and the solution $F(z)$ is assumed bounded and continuous up to the boundary; (b) the functions $a(t)$, $b(t)$, $c(t)$ have finite set $E \subset L$ of discontinuities of jump type and are Hölder – continuous in intervals connecting these discontinuity points, and $F(z)$ has weak power singularities at the points of set E (see, for instance, [1]); (c) the coefficients $a(t)$, $b(t)$ are continuous at finite points of L and have two-side curling of power order $\rho < 1$ at infinity point, and F is bounded in D (see [2], [3], [4]); the case of coefficients with two-side curling of logarithmic order at infinity point is investigated in [5].

We study the problem for the case where the coefficients have a countable set of discontinuities $E = \{t_j, j = 0, \pm1, \pm2, \pm3, \dots\} \subset L$ and two - side curling at infinity simultaneously. We obtain general solution and investigate the conditions of solvability under assumption that the Gakhov index of the problem has singularity either of exponential order less than 1 or of logarithmic order. We investigate the influence of continuous and jump components of function $\nu(t) = \arg[a(t) - ib(t)]$ on solvability of the problem. Here $\nu(t)$ is a branch selected on each interval of continuity of coefficients in such a way that values $\delta_j = \nu(t_j + 0) - \nu(t_j - 0)$ satisfy condition $0 \leq \delta_j < 2\pi$. The results are partially announced in [4].

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Controllability of interconnected evolution equations

Let X_1, X_2 be Hilbert spaces. Consider the control evolution equation

$$\dot{x}_1(t) = A_1 x_1(t) + b_1 v(t), x_1(0) = x_1^0, x_1(t), x_1^0 \in X_1, b_1 \in X_1, v(t) \in \mathbf{R}, \tag{1.1}$$

where $v(t) = (c, x_2(t)), c \in X_2$ and $x_2(t)$ is a mild solution of the control evolution equation

$$\dot{x}_2(t) = A_2 x_2(t) + b_2 u(t), 0 \leq t < +\infty, x_2(0) = x_2^0, x_2(t), x_2^0, b_2 \in X_2 \tag{1.2}$$

Here the linear operators A_1 and A_2 generate strongly continuous C_0-semigroup $S_1(t)$ in X_1 and $S_2(t)$ in X_2 correspondingly.

Definition. Interconnected system (1.1)–(1.2) is said to be exact null-controllable on $[0, t_1]$ if for each $x_1^0 \in X_1$ there exists a square integrable control $u(\cdot) \in L_2[0, t_1]$ such that a mild solution $x\left(t, x_{1,}^0 v(\cdot)\right)$ of equation (1.1) with initial condition x_0^1, satisfies $x_1\left(t_1, x_{1,}^0 v(\cdot)\right) = 0$.

Exact null-controllability conditions for linear control system consisting of two serially connected abstract control evolution equations (1.1)-(1.2) are presented. Applications to interconnected heat and wave equations are considered.

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SOME PROBLEMS IN THE MODERN THEORY OF TRANSMUTATIONS

Methods of transmutation theory now form an important part of modern mathematics, cf. [1]–[3]. They have many applications to theoretical and applied problems.

Let us just itemize some problems in the modern transmutation theory.

1. Theory of Buschman–Erdelyi transmutations [4]–[5]. This class of operators have many applications to partial differential equations, Radon transform theory and many other problems.

2. Theory of operator convolutions and commuting operators [6]. Transmutations are closely connected with commutants. And if commutants in different spaces of analytic functions are completely described by operator convolution theory of I. Dimovski, commutants in standard spaces like C^k are much more difficult to characterize, it was done only recently.

3. Sonine–Dimovski and Poisson–Dimovski transmutations for hyper–Bessel functions and equations [6]–[7].

4. Sonine and Poisson type transmutations for difference–differential operators of Dunkle type.

5. Applications of transmutations to generalized analytic function theory, cf. [8] and papers of V.V. Kravchenko and S. Torba.

6. Methods of fractional integrodifferentiation and integral transforms with special function kernels [7],[3]. In this field let us mention a composition method to derive many classes of transmutations [9].

7. Unitary Sonine–Katrakhov and Poisson–Katrakhov transmutations [4]–[5], [9].

8. Applications to partial differential equations with singularities [3]–[5], [9]–[10].

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The John–Nirenberg constant of BMO^p

The sharp constants in the John–Nirenberg inequality for BMO,

$$\frac{1}{|Q|}|\{t \in Q : |\varphi(t) - \tfrac{1}{|Q|}\int_Q \varphi| \geq \lambda\}| \leq C_1 e^{-c_0\lambda/\|\varphi\|_{\mathrm{BMO}}}, \forall \text{ interval } Q,$$

depend on the choice of the norm; of principal interest is the constant c_0 (this constant is also the supremum of the set of all c such that $e^{c\varphi/\|\varphi\|}$ is guaranteed to be an A_2 weight). For the L^p-based BMO, $c_0 = c_0(p)$ has proved difficult to compute. Until recently, the only results in this direction were those by Korenovskii ($c_0(1) = 2/e$) and Vasyunin ($c_0(2) = 1$); however, their methods do not work for other p. This difficulty was recently overcome (so far, for $p \in [1, 2]$) through the dual formulation of estimating from below the BMO^p norms of logarithms of A_∞ weights. The main result is as follows:

$$c_0(p) = \left[\frac{p}{e}\Big(\Gamma(p) - \int_0^1 t^{p-1}e^t\,dt\Big) + 1\right]^{1/p},$$

and this constant is attained in the inequality above.

I will discuss the method of proof and the connections of this result with the distance in BMO to L^∞ and the $L^\infty \to$ BMO norms of singular integrals.

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Mathematics teaching in trade schools after 1951 reform

Reform of the school system conducted at the beginning of the fifties in Poland introduced new organizational structure of the trade schooling. Schools brought into being in interwar period were replaced by new ones: schools of professional training, basic professional schools and technical colleges. Reconstruction of professional schooling, as many others undertakings of those period, was supposed to support the construction of socialist system based on planned national economy. Through preparing suitable staff, professional schooling was expected to perform important role in industrialization of the country, in modernization of agriculture and in constructing of communication system. Organizational reform in professional schooling was accompanied by programme reform. New systematics of professions and new professional qualification obtained after state exam were introduced. While creating new teaching plans, in most of subjects different presumption of teaching was assumed. Lack of mathematics in professional training schools teaching plans was a great astonishment. Learning on Poland, Polish language and knowledge on production were taught in those schools. Mathematics was taught in basic professional schools. In the framed teaching plan for those schools mathematics was placed in the group of professional and supporting subjects as well as professional technology, classification of materials, professional drawing and physics. Teaching the subject was guided by the rules of practice and usefulness. The main aim of teaching was to extend and deepen fundamentals received in primary school. For mathematics teaching in the first class four hours was assigned, in the second - it was two hours. Efficiency in operations on numbers and decimal fractions was taught as well as proportions, percentage and might calculations, fractions and equations. In technical colleges different attitude to mathematics teaching was observed. Mathematics was supposed to provide pupils with knowledge and skills necessary to understand organization and technology of work as well as modern industrial technologies. Mathematics was also considered as an important selection tool which influenced professional qualifications obtained while state exams.

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Embedding Theorem of different metrics in Morrey spaces

We obtain the embedding theorem in Morrey spaces in terms of the best approximation of function f by means of entire functions of exponential type.

Let $1 \le p \le +\infty$ and $0 \le \lambda \le \frac{1}{p}$. We say that Lebesgue measurable function $f(x)$, $x \in \mathbb{R}$, belongs to Morrey space $M_p^\lambda(\mathbb{R})$ [1], if the norm

$$\|f\|_{M_p^\lambda(\mathbb{R})} = \sup_{\substack{x\in\mathbb{R} \\ r>0}} \left\{ r^{-\lambda} \|f\|_{L_p(x-r,x+r)} \right\}$$

is finite. By $\mathfrak{M}_{\nu p}$ we denote the set of entire functions from $L_p(\mathbb{R})$, $1 \le p \le +\infty$, of exponential type ν.

By $E_\nu(f)_{L_p(\mathbb{R})}$ we denote the best approximation of function f by means of entire functions of exponential type in $L_p(\mathbb{R})$, i.e.

$$E_\nu(f)_{L_p(\mathbb{R})} = \inf \left\{ \|f - g_k\|_{L_p(\mathbb{R})} \colon \ g_k \in \mathfrak{M}_\nu, \ k \le \nu \right\}.$$

Proposition. Let $1 \le p < q \le +\infty$, $0 \le \lambda \le \frac{1}{q}$. Then

$$\sup_{g_\nu \in \mathfrak{M}_{\nu p}} \frac{\|g_\nu\|_{M_q^\lambda(\mathbb{R})}}{\|g_\nu\|_{L_p(\mathbb{R})}} \asymp \nu^{\lambda+\frac{1}{p}-\frac{1}{q}} \quad \text{for all } \ g_\nu \in \mathfrak{M}_{\nu p}.$$

Now we have the following result.

Theorem. Let $1 \le p < q < +\infty$, $0 \le \lambda \le \frac{1}{q}$. Let $f \in L_p(\mathbb{R})$. If the series

$$\sum_{k=1}^{+\infty} k^{\lambda+\frac{1}{p}-\frac{1}{q}-1} E_k(f)_{L_p(\mathbb{R})}$$

is convergent, then $f \in M_q^\lambda(\mathbb{R})$ and the following inequality

$$\|f\|_{M_q^\lambda(\mathbb{R})} \le c_{pq\lambda} \left\{ \|f\|_{L_p(\mathbb{R})} + \sum_{k=1}^{+\infty} k^{\lambda+\frac{1}{p}-\frac{1}{q}-1} E_k(f)_{L_p(\mathbb{R})} \right\}$$

holds. Here $c_{pq\lambda} > 0$ depends only on p, q, λ.

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Conformally invariant higher order operators on the sphere

We classify and explicitly describe conformally invariant differential operators that act upon sections of associated bundles to higher spin representations on the sphere S^n. A higher spin representation is an irreducible representation of SO(n) with highest weight of the form $\lambda = \left(k_1 + \frac{1}{2}, k_2 + \frac{1}{2}, \ldots, \pm\frac{1}{2}\right)$, which we abbreviate to $(k_1, k_2, \ldots)'$, with possible dropping the zero terms. The case $\lambda = (0)'$ corresponds to the Dirac operator, the case of $\lambda = (1)'$ to the Rarita-Schwinger operator. It is known [2] that for any higher spin representation, there is a first order conformally invariant operator (Stein-Weiss gradient) acting on it, we call them higher spin Dirac operators. Using a general result of [3] we prove that there is a (unique up to multiplication) conformally invariant operator of every odd order. Moreover, using the spectrum generating method of [1], we construct explicit formulas for them in an arbitrary order. For $\lambda = (k)'$ they come in the form of a product

$$D \prod_{s=1}^{S} \left(D^2 - s^2 \operatorname{Id} - \frac{16s^2(N-k)}{N(N^2-(2s)^2)} T^* T \right),$$

where D is the higher spin Dirac operator on $(k)'$, T and T^* are other Stein-Weiss gradients and $N = n + 2k - 2$.

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Mixed problems for the Telegraph Equation in the Case of a System Consisting of N Segments with different Densities and Elasticities

We consider the mixed problems for the longitudinal vibrations of a rod governed by the telegraph equation with the Dirichlet and Neumann boundary conditions. The rod consists of n segments with different densities and elasticities.

The rod has the linear density $\rho_1 = const$ and Young's modulus $k_1 = const$ on the segment $0 \leq x \leq l_1$. The rod has the linear density $\rho_i = const$ and Young's modulus $k_i = const$ on the segment $l_{i-1} \leq x \leq l_i$. The rod has the linear density $\rho_n = const$ and Young's modulus $k_n = const$ on the segment $l_{n-1} \leq x \leq l_n = l$.The impedances of these n segments are equal to each other.

The study of the vibrations of the rod subject to the given boundary conditions at its ends is reduced to finding the solution u(x, t) of the following mixed problem for the discontinuous telegraph equation with $a_i = \sqrt{\frac{k_i}{\rho_i}}$

$$u_{tt} = \left\{ a_i^2 u_{xx}(x,t) + c^2 u(x,t) \text{in } Q_2 = [l_{i-1} \leq x \leq l_i] \times [0 \leq t \leq T], \right. \quad (1)$$

with zero initial conditions

$$u\,(x,0) = 0, \quad u_t\,(x,0) = 0, \quad (2)$$

with junction conditions at the joint points $l_i, i = 1 \dots n$,

$$u\,(l_i - 0, t) = u\,(l_i + 0, t)\,, \quad k_{i-1} u_x\,(l_i - 0, t) = k_{i+1} u_x\,(l_i + 0, t) \quad (3)$$

and with boundary conditions:

$$u\,(0,t) = \mu(t), \quad u\,(l,t) = \nu(t), \quad (4^1)$$

(displacements at both ends).

Further, the paper presents a practical application of the explicit formulas for the solution of the above mixed problems for solving and modeling seismic migration problems in a dispersion medium.

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Common Fixed Point Theorem for Occasionally Weakly Compatible (O.W.C.) Mapping in Fuzzy Metric Space Using Rational Inequality

The aim of this paper to prove some Common Fixed Point Theorems in Fuzzy Metric Space with extend the result established in [Common Fixed Point Theorems in Fuzzy metric Space Using Rational Inequality Journal of Advanced Studies in Topology ISSN-2009-388x Vol 2, No. 2, 2011, 46-52].

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A game which incidently teaches algebra.

Children have many fundamental problems with learning algebra. It is also difficult for teachers to carry their students through the threshold of algebraic symbols and operations. On the other hand most of our students love computer games and they can spend a lot of time on playing. Is it possible to overcome difficulties in learning Maths by computer application? With a teacher keeping an eye on his students? DragonBox is an example of such a game. It unnoticeably takes children into algebraic world of symbols. They use colour pictures to place them into different places according to the clear rules. Then very slowly, it replaces pictures by numbers and letters. It encourages students to be active, forces to pay attention, to come back and look for the best solutions. After this amusing entrancetothebasic equations, students can start solving equations on a piece of paper, but then the symbols: letters, numbers, operation signs dont frighten them. Algebraic rules are now familiar game rules. As a Maths teacher I decided to try out DragonBox as a tool for helping students with difficulties in mastering the solving of simple equations. In my contribution I'll show videos recorded in the classroom and talk about my observations and my impression. I'll also put important questions concerning this way of learning algebra as a game with rules consisting on bare manipulation of symbols.

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To anisotropic plane elasticity

It is considered the Lame system

$$a_{11}\frac{\partial^2 u}{\partial x^2} + (a_{12}+a_{21})\frac{\partial^2 u}{\partial y^2} + a_{22}\frac{\partial^2 u}{\partial y^2} = 0 \qquad (1)$$

for a displacement vector $u = (u_1, u_2)$ where

$$a_{11} = \begin{pmatrix} \alpha_1 & \alpha_6 \\ \alpha_6 & \alpha_3 \end{pmatrix}, \quad a_{12} = \begin{pmatrix} \alpha_6 & \alpha_4 \\ \alpha_3 & \alpha_5 \end{pmatrix}, \quad a_{21} = \begin{pmatrix} \alpha_6 & \alpha_3 \\ \alpha_4 & \alpha_5 \end{pmatrix}, \quad a_{22} = \begin{pmatrix} \alpha_3 & \alpha_5 \\ \alpha_5 & \alpha_2 \end{pmatrix}.$$

The characteristic equation $\det[a_{11} + (a_{12}+a_{21})z + a_{22}z^2] = 0$ has two roots ν_1, ν_2 in the upper half-plane. Let

$$J = \begin{pmatrix} \nu_1 & 0 \\ 0 & \nu_2 \end{pmatrix}, \ \nu_1 \neq \nu_2; \quad J = \begin{pmatrix} \nu & 1 \\ 0 & \nu \end{pmatrix}, \ \nu_j = \nu.$$

Then there exists [1] an invertible matrix $b \in \mathbb{C}^{l\times l}$, such that

$$a_0 b + a_1 bJ + a_2 bJ^2 = 0, \quad \det \begin{pmatrix} b & \overline{b} \\ bJ & \overline{bJ} \end{pmatrix} \neq 0.$$

If a matrix b_1 also satisfies these conditions then $b_1 = bd$ where the matrix d is invertible and commutes with J. Hence the matrix- valued function $H(\xi) = \operatorname{Im}[b(-\xi_2 + \xi_1 J)(\xi_1 + \xi_2 J)^{-1} b^{-1}]$, $\xi = (\xi_1, \xi_2) \in \mathbb{R}^2$, doesn't depend on b.

Let a domain D be bounded by a Lyapunov contour Γ. The generalized potential of double layer is defined by the integral

$$u(z) = \frac{1}{\pi} \int_\Gamma Q(t, t - z)\varphi(t)|dt|, \quad z \in D,$$

where $Q(t, \xi) = |\xi|^{-2}[n_1(t)\xi_1 + n_2(t)]H(\xi)$ and $n = n_1 + in_2$ implies the unit outer normal. If the vector- valued function $\varphi \in C(\Gamma)$ then $u \in C(\overline{D})$ satisfies (1) in the domain D.

Using this potential the Dirichlet problem in $C(\overline{D})$ may be reduced to the equivalent Fredholm integral system on the boundary Γ. The analogues result is also valid for the Neumann problem. Its boundary condition can be written in the Dirichlet form with respect to a so called conjugate function v. For this function the generalized potential of double layer is defined as above by substituting b into $c = a_{21} b + a_{22} bJ$. The matrixes b, c can be found explicitly [2] and the corresponding expressions can be also received for kernels Q.

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On Nonlinear Operators, Fixed-Point and Solvability of Nonlinear Equations

In the present paper we consider the boundary-value problem for the fully nonlinear equation of the second order

$$F(x, u, Du, \Delta u) = h(x), \quad x \in \Omega, \tag{1.1}$$

and also for the nonlinear equations with p-Laplacian that depend upon the parameters λ and μ on the smooth bounded domain $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$. Equations of such type can arise in the diffusion processes, reaction-diffusion processes etc. of the steady-state case. We study the solvability of these problems with use of the general results of such type as in [2] ([1]) that allow us to study the posed problems under more general conditions. We discuss here on the nonlinear continuous mappings acting in a Banach space and an equation (inclusion) with mappings of such type. Moreover we lead here a fixed-point theorem for nonlinear continuous mappings on Banach spaces and a solvability theorem for the nonlinear equations with continuous operators which can used for study various problems. Main theorem of the present article is a generalization of Lax-Milgram theorem to the result for the nonlinear operators acting on Banach spaces.

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Clifford superanalysis

We first introduce some basic concepts about superspace and superanalysis using the Vladimirov-Volovich approach. Then we introduce the basics for Clifford analysis in superspace, leading to the expected ortho-symplectic structure. Next we recall what we think should be the correct rules for differential forms on superspace and explain how one can integrate superforms. This leads to a way to introduce the Berezin integral and, more in general, to integration in "deepspace". All this can be applied to Clifford-superforms.

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On Florencio-Paúl-Virués diagonal theorem

Miscellaneous versions of the diagonal theorem stand for abstract forms of the sliding-hump method and have been investigated by many authors. An interesting complement of the diagonal theorem was given by M. Florencio, P. J. Paúl and J. M. Virués in [3].

The diagonal theorem is applied efficiently in measure theory and functional analysis allowing, in particular, more general formulations of series of classical theorems (e.g. Banach-Steinhaus, Nikodym, Orlicz-Pettis, Vitali-Hahn-Saks, Hahn-Schur, and Bessaga-Pełczyński theorems). The diagonal techniques seem to be more general than the standard Baire category methods (cf. [2]). There are also important applications of the diagonal theorem in the theory of distributions. In particular, the theorem is a basic tool in an elementary proof of the functional and sequential approaches to theory of distributions (cf. [1], [4]).

We present the Florencio-Paúl-Virués diagonal theorem formulated for quasi-normed groups and essentially complete the proof given in [3].

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On the local uniqueness for weakly hyperbolic equations

In the first part of the talk we shall discuss some old and recent results concerning the local (in space) uniqueness for a weakly hyperbolic Cauchy problem. Then we shall prove, in some special cases, a more quantitative form of local uniqueness, by constructing a local (in space) energy for which an apriori estimate holds.

This allows us to get the local version of the analytic propagation for sufficently smooth solutions.

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Geodesic completeness of Lorentzian manifolds of low regularity

We propose a definition of geodesic completeness for Lorentzian manifolds of low regularity based on the geometric theory of generalized functions ([1]). As a physically relevant example we prove completenss of a class of impulsive gravitational wave space-times of the form $M = N \times \mathbb{R}^2_1$, where (N, h) is a Riemannian manifold of arbitrary dimension and M carries the line element $ds^2 = dh^2 + 2dudv + f(x)\delta(u)du^2$ with dh^2 the line element of N and δ the Dirac measure ([2]).

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On some solutions of certain versions of the models: "sigma" and baby Skyrme ones

The so-called "sigma" model is very important field model, some version of it describes static Heisenberg ferromagnet [4]. Another interesting field model is the so-called baby Skyrme model, which is some analogon (on the plane) of the Skyrme model describing baryons as topological solitons [1]. Some solutions of certain versions of the models: "sigma" and baby Skyrme ones, will be presented, among others, by applying some results included in [2], [3].

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Frames, General Fréchet frames, and series representations

Frame theory is one of the basic topics of pure and applied mathematics nowdays. First we give a short introduction on frames in Hilbert and Banach spaces, which motivated our work in Fréchet spaces. Then we direct our attention on projective and inductive limits of Banach spaces (for example, the Schwartz space $\mathcal{S}(\mathbb{R}^n)$ of rapidly decreasing functions and its dual, the space of tempered distributions $\mathcal{S}'(\mathbb{R}^n)$) and General Fréchet frames for such spaces.

Hilbert frames extend orthonormal bases and still allow series representations of every element in the space. However, the natural extension of Hilbert frames to Banach spaces do not necessarily lead to expansions in Banach spaces and there have been investigations in this direction. Now our attention goes to projective and inductive limits of Banach spaces. First, we introduced and investigated Fréche frame. Recently, we introduced more general sequences, called General Fréche frames, and investigated the question for series representations via such frames. In the talk we will present some results on General Fréche frames. We will give examples of General Fréche frames which are not Fréche frames and thus will show that the concept *General Fréche frame* is an essential extension of the concept *Fréche frame*.

The new results are joint work with Stevan Pilipović.

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The exponential map of $C^{1,1}$-metrics

Many essential properties of (semi-) Riemannian manifolds depend on the existence of Riemannian normal coordinates, i.e., on the fact that the exponential map is a local diffeomorphism. In this talk we address the question of the regularity of the exponential map for manifolds of lower, namely $C^{1,1}$, regularity. For the case of Riemannian or Lorentzian manifolds we show that the exponential map is still a bi-Lipschitz homeomorphism. Our methods of proof are based on regularization, combined with results from Riemannian and Lorentzian comparison geometry. In addition, we obtain existence of convex neighborhoods in both cases. Finally, we indicate implications of these results to causality theory in general relativity.

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TBA type equations and tropical curves

We revisit the wall-crossing behaviour of solutions of a class of Thermodynamic Bethe Ansatz type integral equations, expressed as sums of “instanton corrections”. We explain how a set of tropical curves (with signs) emerges naturally from each instanton correction, then show that the weighted sum over all such curves is in fact a tropical count. This goes through to the q-deformed setting. This construction can be regarded as a formal mirror-symmetric statement in the framework proposed by Gaiotto, Moore and Neitzke. Joint work with S. A. Filippini.

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Quaternionic functions as tools for the classification of orthogonal complex structures

The classification of orthogonal complex structures on dense open subsets Ω of the Euclidean space $\mathbb{H} = \mathbb{R}^4$ is an open problem studied by experts in differential geometry. In the case $\Omega = \mathbb{H} \setminus \Lambda$ where Λ is a line, a circle or a set whose 1-dimensional Hausdorff measure vanishes, it has been solved in [3, 4] with the aid of the conformal transformations of $\mathbb{H} \cup \{\infty\} \cong \mathbb{HP}^1$. No instrument was available to study other cases, not even that of $\mathbb{H}$ minus a parabola.

The theory of regular quaternionic functions introduced in [2] provides the tools for a new approach to the problem. Indeed, new orthogonal complex structures can be constructed pushing forward the standard structure of $\mathbb{H} \setminus \mathbb{R}$ via regular functions. Moreover, regular functions lift to the twistor space $\mathbb{CP}^3$ and the resulting lifts can be used to undertake a classification. Both of these results are included in the paper [1].

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A vector fields approach to smoothing and decaying estimates for equations in anisotropic media

It is well known that the vector fields

$$\Omega = x \wedge D = (\Omega_{ij})_{i<j}\,, \qquad \Omega_{ij} = x_i D_j - x_j D_i$$

commute with the Laplacian $-\Delta$. Hence we have

$$Pu = f \quad \Rightarrow \quad P(\Omega u) = \Omega f,$$

where $P = p(D_t, -\triangle)$, and in this way we can control the growth/decaying order of solution u to the equation $Pu = f$. This fact was actually used to induce some decaying estimates for the wave equation and smoothing estimates for the Scrödinger equation. In this talk, we will discuss how to trace this idea for equations with the Laplacian $-\Delta$ replaced by general elliptic (pseudo-)differential operators. Such situation naturally arises in the equation of linear elasticity for crystals and Maxwell equations in anisotropic media. In general, elliptic operators do not always have corresponding vector fields which commute with them, but some useful lemma and its application will be stated.

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Conservation laws with discontinuous flux

We are interested in a scalar conservation law in an arbitrary dimension d with a discontinuous flux F

$$\begin{aligned} u_t + \operatorname{div} F(x,u) &= 0 \quad \text{in } (0,\infty)\times\mathbb{R}^d, \\ u(0,\cdot) &= u_0 \quad \text{in } \mathbb{R}^d. \end{aligned} \tag{1}$$

The flux is supposed to be a discontinuous function in the spatial variable x and in an unknown function u. Under some additional hypothesis on the structure of possible discontinuities, we formulate an appropriate notion of entropy solution and establish its existence and uniqueness. The structure of the flux function corresponds to the one proposed by [4] for the case of fluxes discontinuous in x. The framework for proving the existence and uniqueness of entropy weak solutions is provided by the studies on entropy measure-valued solutions, cf. [3] and may be viewed as a corollary of the uniqueness theorem for entropy measure-valued solutions. Moreover, the techniques using comparsion principle will be presented.

There are numerous applications of considerations on discontinuous flux such as sedimentation process, two phase flow in porous media, modeling of traffic flow and others. The studies are also motivated by an implicit constitutive theory. The understanding of scalar hyperbolic conservation laws with a discontinuous (or multi-valued) flux represent a good starting point for a progress in the mathematical theory for evolutionary problems of elasticity with implicit or discontinuous relations between the Cauchy stress and the deformation gradient.

The talk is based on joint works with M. Bulíček, P. Gwiazda and J. Málek, [1, 2] and also on the work in progress with P. Wittbold and A. Zimmermann.

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Semi-reflection as a method for developing mathematical knowledge for teaching

The understanding of basic concepts in advanced mathematics by the students - in our case future mathematics teachers - has been subjected very often to criticism. The research related to the perception of these concepts and their use in problem solving has lead to identify a number of phenomena in the field of understanding. It is noteworthy that such research is usually led by experienced didacticians and mathematicians who analyze the problem from their own point of view. In our research we employed a different method. We have decided to extend our own observations by the students' point of view. The students were not asked to explain their own ways of reasoning or interpret their own solutions; their role was to interpret the strategies and behaviors of their peers - usually participants of the same organized didactical process.

The research aim was to examine the understanding of the concept of function limit. Since this topic was extensively analyzed in the literature, it gave us the opportunity to compare the results of our observation with the results which did not included the students' observations and opinions in the process of analysis. Multistage approach to the problem of understanding the concept of the limit reaped the following benefits:

1. It confirmed the similarities of strategies and the degree of understanding of the concepts with those described in the literature.
2. It gave an opportunity to emphasize the students' opinions on the topic.
3. It enabled the detection and justification of the ways of studying advanced mathematics in courses offered to future mathematics teachers.
4. It supplied didactical material which is much useful in teacher education by extending it with mathematical knowledge for teaching.

The last point shows that the conducted research by the use of the described methodology can be directly used in mathematics teacher education.

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Global existence for semilinear wave equations with the blow-up term in high dimensions

We are interested in the fact that the lifespan $T(\varepsilon)$, the maximal existence time, of a classical solution of

$$\left\{ \begin{array}{l} u_{tt} - \Delta u = u^2 \quad \text{in} \quad \mathbf{R}^4 \times [0,\infty), \\ u(x,0) = \varepsilon f(x), u_t(x,0) = \varepsilon g(x) \end{array} \right. \tag{1}$$

of a small parameter $\varepsilon > 0$, compactly supported smooth functions f and g, has an estimate

$$\exp\left(c\varepsilon^{-2}\right) \le T(\varepsilon) \le \exp\left(C\varepsilon^{-2}\right), \tag{2}$$

where c and C are positive constants depending only on f and g. This result is due to Li and Zhou [1] for the lower bound and to Takamura and Wakasa [2] for the upper bound. We note that its importance is extremely huge as the problem is related to the final open part of Strauss' conjecture on semilinear wave equations as well as one of the last open optimality of the general theory for nonlinear wave equations.

In this talk, I would like to introduce you the following theorems.

Theorem 1. (2) still holds even if u^2 in (1) is replaced by

$$u(x,t)^2 - \frac{1}{\pi^2}\int_0^t d\tau \int_{|\xi|\le 1} \frac{(u_t u)(x+(t-\tau)\xi,\tau)}{\sqrt{1-|\xi|^2}}d\xi - \frac{\varepsilon^2}{2\pi^2}\int_{|\xi|\le 1}\frac{f(x+t\xi)^2}{\sqrt{1-|\xi|^2}}d\xi. \tag{3}$$

Theorem 2. $T(\varepsilon) = \infty$ holds if u^2 in (1) is replaced by

$$u(x,t)^2 - \frac{1}{2\pi^2}\int_0^t d\tau \int_{|\omega|=1} (u_t u)(x+(t-\tau)\omega,\tau)dS_\omega - \frac{\varepsilon}{4\pi^2}\int_{|\omega|=1}(\varepsilon f^2 + \Delta f + 2\omega\cdot\nabla g)(x+t\omega)dS_\omega. \tag{4}$$

We are looking for a criterion to get the global in time existence of a solution, $T(\varepsilon) = \infty$, for more general terms of the "critical" power in four space dimensions. Except for this situation, there is no possibility to discuss this kind of problems for "classical" solutions in high dimensions.

All the results in this talk are based on joint works with Kyouhei Wakasa. The speaker is partially supported by the Grant-in-Aid for Scientific Research (C)(No.24540183), Japan Society for the Promotion of Science.

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Almost sure global wellposedness for the periodic derivative NLS

In recent years, a study on a low regularity global in time solution for the dispersive nonlinear equations has been developed by several methods. Based on various smoothing effects, local wellposedness results have been established. In order to establish global existence results, one need to control the energy exchanges between low and high frequencies, for example by Fourier truncation method and I-method. We exhibit the subsequent approach to the global in time existence described via Gibbs measure for the periodic derivative NLS equation [1, 3].

Consider the initial value problem for the derivative nonlinear Schrödinger equation on the torus

$$\partial_t u - i\partial_x^2 u = iu^2\partial_x \overline{u} + iu\frac{1}{\pi}\int_0^{2\pi} \mathrm{Im}(u\overline{u}_x)(t,x)\,dx + (\text{Cubic-Quintic nonlinear terms})$$

and ensemble initial data to be random Fourier series of the form

$$u(0,x,\omega) = \phi(x,\omega) = g_0(\omega) + \sum_{k\in\mathbb{Z}\setminus\{0\}} \frac{g_k(\omega)}{k} e^{ikx} \in H^{1/2-\varepsilon}\setminus H^{1/2}$$

for $\varepsilon > 0$ and almost all $\omega \in \Omega$, where $\{g_k\}$ are independent complex Gaussian variables on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

We propose an approach to overcome the lack of local wellposedness, and prove global wellposedness almost surely for the data in the support of the Gaussian measures on $H^s \cap \mathcal{F}L^{1/2,\infty}$, $s < 1/2$, where $\mathcal{F}L^{s,p}$ is some auxiliary function space. This result extends the one in [2], which assumes $s \geq 1/2$.

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On definitions of general local and global Morrey-type spaces

One of popular definitions of general local Morrey-type spaces is as follows. Let $B(x,r)$ denote the open ball in $\mathbb{R}^n$ centered at $x \in \mathbb{R}^n$ of radius $r > 0$. Let $0 < p, \theta \leq \infty$ and let $w \in \Omega_\theta$ i. e. let w be a non-negative Lebesgue measurable function on $(0,\infty)$, not equivalent to 0 on (t,∞) for any $t > 0$, and such that $\|w\|_{L_\theta(t,\infty)} < \infty$ for some $t > 0$.

The local Morrey-type space $LM_{p\theta,w(\cdot)}$ is the space of all functions f Lebesgue measurable on $\mathbb{R}^n$ with finite quasi-norm

$$\|f\|_{LM_{p\theta,w(\cdot)}} = \big\| w(r)\|f\|_{L_p(B(0,r))} \big\|_{L_\theta(0,\infty)}.$$

Theorem. *If $\theta < \infty$ and $w \in \Omega_\theta$, then for each $\varepsilon > 0$ there exists a function $w_\varepsilon \in \Omega_\theta$ such that $w_\varepsilon \geq w$ on $(0,\infty)$, $w_\varepsilon > 0$ on $(0,\infty)$, $LM_{p\theta,w_\varepsilon(\cdot)} = LM_{p\theta,w(\cdot)}$, and*

$$\|f\|_{LM_{p\theta,w(\cdot)}} \leq \|f\|_{LM_{p\theta,w_\varepsilon(\cdot)}} \leq (1+\varepsilon)\|f\|_{LM_{p\theta,w(\cdot)}}$$

for all $f \in LM_{p\theta,w(\cdot)}$.

If $\theta = \infty$ and $w \in \Omega_\infty$, then there exists a function $\widetilde{w} \in \Omega_\theta$ such that $\widetilde{w} \geq w$ on $0, \infty)$, $\widetilde{w} > 0$ on $(0, \infty)$, $LM_{p\infty,\widetilde{w}(\cdot)} = LM_{p\infty,w(\cdot)}$, and

$$\|f\|_{LM_{p\infty,\widetilde{w}(\cdot)}} = \|f\|_{LM_{p\infty,w(\cdot)}}$$

for all $f \in LM_{p\infty,w(\cdot)}$.

A similar statement holds for global Morrey-type spaces $GM_{p\theta,w(\cdot)}$.

For proofs and applications see [1].

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Extremal problems for partially non-overlapping domains on equiangular system points.

Let $r(B; a)$ – inner radius domain $B \subset \overline{\mathbb{C}}$ with respect to a point $a \in B$.

This work a study the following problem.

Problem. Let $n \in \mathbb{N}$, $n \geq 2$. Maximum functional be found

$$\left(r\left(B_0\,;\,0\right) \cdot r\left(B_\infty\,;\,\infty\right)\right)^\gamma \cdot \prod_{k=1}^{n} r\left(B_k\,;\,a_k\right),$$

where $A_n = \{a_k\}_{k=1}^n$ – arbitrary n-equiangular system points and $\{B_0, \{B_k\}_{k=1}^n, B_\infty\}$ – arbitrary set partially non-overlapping domains, $0 \in B_0$, $\infty \in B_\infty$, $a_k \in B_k$, and all extremal the describe $k = \overline{1, n}$.

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Main motivation tendencies of future mathematics' teachers' educational and professional activity

Improving the quality of future mathematics' teachers' professional training demands the development of their motivation.

Motives of educational and professional activity can be combined into two interrelated groups: perspective-stimulating and intellectually-stimulating motives.

Perspective-stimulating motives are based on the understanding of professional prestige in the society, importance of professional skills and habits, in particular: realization of the subject's importance and application, connection of the subject with future profession; expectation to get the profit of the future; to develop the feeling of responsibility.

Intellectually-stimulating motives are based on obtaining pleasure from the condition process; presence of interest to certain professional activity, curiosity, desire to self-development in certain professional field.

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Performance evaluation of queueing systems with random capacity demands and bounded buffer space

We investigate non-classical queueing systems with demands characterized by some random capacity ζ under assumption that demand service time ξ generally depends on its capacity [1]. This dependency can be defined by the joint distribution function $F(x,y) = \mathbf{P}\{\zeta < x, \xi < x\}$. The total demands capacity $\sigma(t)$ (i.e. the sum of capacities of demands present in the system at time instant t) can be bounded by some constant value V that is named buffer space volume. Such systems have been used to model and solve the various problems occurring in the design of computer and communicating systems.

The main steady-state performance characteristics of such systems is loss probability P_{loss} (the relative part of demands that was lost in the system during infinite time interval) and probability of loss of capacity unit Q_{loss} [2] (the relative part of total demands capacity that was lost during infinite time interval).

In our presentation we determine these characteristics for some systems with bounded buffer space and compare the results of our calculations with the results of calculations of loss probability in classical regenerative queueing systems using the relation obtained in [3]. Numerical examples and results of simulation are attached as well.

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Recovery opreator of periodic functions

Let (X,Y) be pair of functional spaces of 1-periodic functions, X embedded in $C[0,1]^n$. Our aim is to find the nodes $\{t_k\}_{k=1}^M$ and functions $\{\phi_k(x)\}_{k=1}^M$, such that the error

$$\delta_M(X,Y) = \sup_{\|f\|_X=1} \|f_\lambda - \sum_{k=1}^M f(t_k)\phi_k(x)\|_Y$$

will be minimal when order M increase.

The problem of recovering of function from the classes with dominant mixed derivative is considered in many works.

The aim of this talk is to construct a recovery operator for which the error coincides with the order of corresponding orthodiameter:

$$d_M^{\perp}(X,Y) = \inf_{\{g_j\}_{j=1}^M} \sup_{\|f\|_X=1} \|f - \sum_{j=1}^M (f,g_j)g_j\|,$$

here the exact lower bound is taken over all orthogonal systems $\{g_j\}_{j=1}^M$ from $L_\infty[0,1]^n$.

For a function $f \in C[0,1]^n$ we define the transform

(1) $$F_m(f;x) = \sum_{\substack{\psi(k)=m \\ k\in\mathbb{N}^n}} \frac{1}{2^{|k|}} \sum_{0\le r<2^k} f(\frac{r}{2^k})\phi_{k,r}(x+\frac{r}{2^k}),$$

(2) $$\phi_{k,r}(x) = \sum_{0\le\nu\le k} (-1)^{\sum_{j=1}^{n-1}(r_j+1)sgn(k_j-\nu_j)} \sum_{\mu\in\rho(\nu)} e^{2\pi i\mu x}.$$

Here $\mu x := \sum_{j=1}^n \mu_j x_j$, $|k| := k_1 + \ldots + k_n$, $\rho(\nu) = \{\mu = (\mu_1, ..., \mu_n) \in \mathbb{N}^n : \left[2^{\nu_j-2}\right] \le |\mu_j| < 2^{\nu_j-1}\}$, $[x]$ is integer part of x, and $\nu \le \mu$ means that $\nu_j \le \mu_j$, $j = \overline{1,n}$.

Theorem. *Let $m \ge \psi(1)$, $F_m(f)$ defined by the relations (1), (2) , M is number of nodes in the definition of $F_m(f)$. If $1 < p \le 2 \le q \le \infty$, $\alpha_0 > \frac{1}{p}$, then*

$$\sup_{\|f\|_{SW_p^\alpha}=1} \|f - F_m(f)\|_{L_q} \sim d_M^\perp(SW_p^\alpha, L_q),$$

$$\sup_{\|f\|_{SH_p^\alpha}=1} \|f - F_m(f)\|_{L_q} \sim d_M^\perp(SH_p^\alpha, L_q).$$

The talk is based on joint work with E.D. Nursultanov (M.V.Lomonosov Moscow State University).

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Modulation spaces, harmonic analysis and pseudo-differential operators

In the present talk we present recent results on composition, continuity and Schatten-von Neumann (SvN) properties for operators and pseudo-differential operators (ΨDOs) when acting on modulation spaces. For example we present necessary and sufficient conditions in order for the Weyl product should be continuous on modulation spaces. Such question is strongly connected to questions wether compositions of ΨDOs with symbols in modulation spaces remain as ΨDOs with a symbol in a modulation space.

We also present necessary and sufficient conditions for ΨDOs with symbols in modulation spaces should be SvN operators of certain degree in the interval $(0, \infty]$. Note here that there are so far only few results in the literature on SvN operators with degrees less than one.

The talk is based on joint works with K. H. Gröchenig, and with E. Cordero and P. Wahlberg.

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Generalized Fourier transform and Pseudo–Differential Operators

We consider the development of the Fourier analysis based on a boundary value problem for the derivative operator on a segment. In particular, we derive an explicit formula for the convolution generated by the problem. Starts an interesting direction of discrete analysis based on elliptic boundary value problems, continuing, in a sense, the analysis on the torus started by M. Ruzhansky and V. Turunen [1]–[2], in which case one may think of a problem having periodic boundary conditions.

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Polyharmonic equation with point interactions

We consider some models of thin flat elastic plates with point interactions. One of the mathematical interpretations: 2D Biharmonic equation in punctured domain. In a Hilbert space it is investigated a class of well-posed problems for Polyharmonic equation in punctured domain. It's taken an analogue of the Green's formula and a class of self-adjoint operators.

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APPLICATIONS OF THE METHOD OF DYNAMIC PROGRAMMING TO INVERSE PROBLEMS OF DYNAMICS

A controlled system is under consideration

$$\frac{dx(t)}{dt} = g(t, x(t)) + f(t, x(t))u(t), \qquad t \in [0, T], \tag{1}$$

where $x \in \mathbb{R}^n$ is a state vector, a control parameter $u \in \mathbb{R}^r$ is restricted

$$u \in U = \{u_i \in [a_i^-, a_i^+], \quad a_i^- < a_i^+, \qquad i = 1, 2, \ldots, r\}. \tag{2}$$

We know a continuous function $y(\cdot) : [0, T] \to \mathbb{R}^n$, which is the measurement of a trajectory $x_*(t)$ of the system (1). It is known that the trajectory $x_*(t)$ belongs to the strip of admissible errors Ω_δ, i.e.$(t, x_*(t)) \in \Omega_\delta = \{(t, x) \in [0, T] \times \mathbb{R}^n : \ ||x - y(t)|| \leq \delta\}$, where $\delta > 0$ is the parameter of measurement errors, the symbol $||z||$ denotes the Euclidean norm of a vector z.

The goal of the inverse problem of dynamic is to construct a control and a trajectory of the system (1), (2), which approximate the unknown trajectory $x_*(\cdot)$ [2].

We introduce an auxiliary optimal positional control problem [1] for the system (1), (2) minimizing the residual functional

$$I(u(\cdot)) = \int_0^T \left[\frac{(x(t) - y(t))^2}{2} + \frac{\alpha}{2} ||u(t)||^2 \right] dt, \tag{3}$$

here $\alpha > 0$ is a small regularization parameter.

We apply the method of dynamic programming [3] in the problem (1)–(3) to construct the trajectories of the system within the strip of admissible statistic errors. We provide the extremal trajectories which minimize the residual functional in the strip Ω_δ.

It is proven that the extremal trajectories approximate the exact solution $x_*(\cdot)$ of the inverse problem of dynamics. The estimates of the approximation are obtained, as $\delta \to 0$, $\alpha \to 0$. Numerical algorithms are created. Results of simulations are exposed.

The work was supported by the Russian Foundation for Basic Researches (projects No. 11-01-00214) and by the Program of Presidium RAS for Basic Researches on Mathematical Control Theory.

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Hypoellipticity and solvability for classes of second order Shubin operators with nonhypoelliptic symbols

We investigate the hypoellipticity and the solvability in $S(\mathbb{R}^n)$ for classes of second order Shubin differential operators $P(x, D)$ with real–valued non-negative principal symbol which is not hypoelliptic symbol. In the case $n \geq 2$ we require that no rotation terms appear, excluding, in particular, the twisted Laplacian.

The first step is the reduction via global normal form transformation of the equation $Pu = f$ to a (generalized) complex Airy type equation perturbed, in the multidimensional case, by the $n-1$ dimensional laplacian on a suitable linear subspace of $\mathbb{R}^n$.
Secondly, we introduce anisotropic weighted spaces $HSC^{s;\overline{r}}(\mathbb{R} \times \mathbb{R}^{n-1})$, $s \in \mathbb{R}$, $r = (r_1, r_2) \in \mathbb{R}^2$. Broadly speaking, such spaces are equivalent to the Shubin weighted spaces, if we restrict to $\mathbb{R}$, while restricted to the second component we are reduced to the weighted spaces used in the study of the SG–pseudodifferential operators, associated in a natural way to the Airy type structure. The choice of the functional frame combined with estimates on Airy type parametrices allows us to derive sharp estimates in the weighted Sobolev spaces and to study the hypoellipticity and the solvability in $S(\mathbb{R}^n)$ of the original operators. Moreover, we describe the spectrum of P and prove estimates of the resolvent operator . We also outline hypoellipticiy and solvability results in the framework of the Gelfand–Shilov spaces.

The talk is based on joint work with T. Gramchev (Università di Cagliari).

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Scattering on Non-Compact Graphs

Differential operators on graphs often appear in physics, physical chemistry, biology etc. We investigate scattering problems on noncompact graphs, containing compact edges.

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Cauchy problem for a first order ordinary differential systems in the plane with variable coefficients

Let $-\infty < t_1 < t_2 < \infty$, $S[t_1, t_2]$ be set of measurable, essentially bounded function $f(t)$ in $[t_1, t_2]$. $W^1_\infty[t_1, t_2]$ is the class of function $f(t)$, for which $f'(t) \in S[t_1, t_2]$. We consider the system

$$u' = f(t)u + g(t)v + h(t), \;\; v' = g(t)u - f(t)v + q(t) \tag{1}$$

in $[t_1, t_2]$, where $h(t)$, $q(t) \in L_1[t_1, t_2]$; $f(t)$, $g(t) \in S[t_1, t_2]$.

In the given work we will construct the general solutions of the system (1) in the class

$$W^1_\infty[t_1, t_2] \cap C[t_1, t_2] \tag{2}$$

Let $t_0 \in [t_1, t_2]$. Let us consider the Cauchy problem: find the solution of system (1) from the class (2) satisfying the Cauchy conditions $u(t_0) = \alpha$, $v(t_0) = \beta$, where α and β are given real numbers. By us problem K is solved:

$$u = \alpha \Re P_1(t) + P_2(t) + \beta \Im P_1(t) - P_2(t) + \Re P(t), \; v = \alpha \Im P_1(t) + P_2(t) - \beta \Re P_1(t) - P_2(t) + \Im P_3(t),$$

where $P_1(t) = \sum\limits_{j=1}^{\infty} I_{2j-1}(t)$, $P_2(t) = 1 + \sum\limits_{j=1}^{\infty} I_{2j}(t)$, $P_3(t) = \sum\limits_{j=0}^{\infty} A_j(t)$, $I_j(t) = \int\limits_{t_0}^{t} b(\tau) I_{j-1}(\tau) d\tau$, $A_j(t) = \int\limits_{t_0}^{t} b(\tau) A_{j-1}(\tau) d\tau$, $(j = \overline{2, \infty})$, $I_1(t) = \int\limits_{t_0}^{t} b(\tau) d\tau$, $A_1(t) = \int\limits_{t_0}^{t} b(\tau) \overline{A_0(\tau)} d\tau$.

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Properties and computation of Born–Jordan related time-frequency distributions

We study properties of functions and tempered distributions by means of certain Cohen class bilinear time-frequency distributions related to the Born–Jordan quantization of pseudo-differential operators. We obtain smoothness and continuity results for the involved transforms, and also present an efficient FFT-based algorithm, computing time-frequency pictures of real-life signals. Advantages of this approach are demonstrated by comparisons to spectrograms.

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Generalized linear differential equations in a Banach space: Continuous dependence on a parameter

In the contribution we present new conditions ensuring the continuous dependence on a parameter k of solutions to linear integral equations of the form

$$x(t) = \widetilde{x}_k + \int_a^t \mathrm{d}[A_k]\, x + f_k(t) - f_k(a)\,, \quad t \in [a,b]\,, k \in \mathbb{N}\,,$$

where $-\infty < a < b < \infty$, X is a Banach space, $L(X)$ is the Banach space of linear bounded operators on X, $\widetilde{x}_k \in X$, $A_k : [a,b] \to L(X)$ have bounded variations on $[a,b]$, $f_k : [a,b] \to X$ are regulated on $[a,b]$. The integrals are understood as the abstract Kurzweil-Stieltjes integrals and the studied equations are usually called generalized linear differential equations (in the sense of J. Kurzweil, cf. [1] or [2]).

Our main theorem concerns the case when the variations $\mathrm{var}_a^b\, A_k$ need not be uniformly bounded and it is an analogy of the Opial's result [5] for ODEs.

Applications to linear dynamic equations on time scales are then enabled by their relationship with generalized differential equations as disclosed by A. Slavík in [6].

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Scalar Riemann-Hilbert problem for circular multiply connected domains

We solve the scalar Riemann-Hilbert problem for circular multiply domains. We discuss the $\mathbb{R}$-linear and the Schwarz problem. We reduce the boundary value problem to functional equations and solve them using the Poincarè series.

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Conformal composition operators on Sobolev spaces and Brennan's conjecture

(Joint work with Vladimir Gol'dshtein)

We study composition operators on Sobolev spaces generated by conformal mappings of plane Euclidean domains $\Omega, \Omega' \subset \mathbb{R}^2$ in connection with Brennan's conjecture. Brennan's conjecture states integrability of the complex derivative φ' of a plane conformal mapping $\varphi : \Omega \to \mathbb{D}$, $\mathbb{D}$ is the unit disc, in the power $4/3 < s < 4$. We prove that Brennan's conjecture holds if and only if φ generates by the composition rule $\varphi^*(f) = f \circ \varphi$, $f \in L^1_p(\mathbb{D})$, $2 < p < \infty$, a bounded composition operator

$$\varphi^* : L^1_p(\mathbb{D}) \to L^1_q(\Omega), \;\; q = ps/(p + s - 2).$$

This result has applications in the weighted Sobolev type embedding theory and degenerate elliptic boundary value problems.

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Some Results of a Weighted Convolution Algebra

In this study, we give another generalization of $A^{p(.)}_{w}(\mathbb{R}^n)$ in [2] to the weighted variable Lebesgue spaces $L^{p(.)}_{\omega}(\mathbb{R}^n)$. We define $A^{p(.)}_{w,\omega}(\mathbb{R}^n)$ to be the space of all complex-valued functions in $L^1_w(\mathbb{R}^n)$ whose Fourier transforms belong to $L^{p(.)}_{\omega}(\mathbb{R}^n)$. Also, we investigate some inclusions and compact embeddings properties and further discuss multipliers of this spaces.

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A partial differential equation with a closed parabolic boundary

In the article [1] the following equation is considered

$$\partial_{\overline{z}}(w + q(z)\overline{w}) = f(z), \quad z \in G, \tag{1}$$

where $w(z)$ is an unknown complex-valued function, $q(z)$ and $f(z)$ are given functions in a bounded simply connected domain G, the boundary of which is denoted by ∂G. It is assumed that $q(z)$, $w(z) \in D_{1,p}(\overline{G})$ ($\overline{G} = G + \partial G$), $f(z) \in L_p(\overline{G})$, $p > 2$, and $\partial G \in C^1_\alpha$, $\alpha > 0$, and moreover

$$|q(z)| = 1 \quad \text{as} \quad z \in \partial G$$

and $q(z) \neq -1$, $z \in \partial G$.

Now we consider the case $q(z_k) = -1$, $z_k \in \partial G$, $k = 1, ..., n$, and also $z_i \neq z_j$ if $i \neq j$. Since $|q(z)| = 1$, $z \in \partial G$, we can write $q(z)$ in the form $q(z) = e^{i\varphi}$, where $\varphi = arg\, q(z)$.

We consider the case that the number

$$\lambda = Ind_{\partial G}\, [e^{i\frac{\varphi}{2}}] = Ind_{\partial G}\, [\sqrt{q(z)}] = \frac{1}{2} Ind_{\partial G}\, [q(z)],$$

is integer (then $Ind_{\partial G}\, q(z)$ is an even number). We obtain the following assertion with respect to solvability of equation (1):

for $\lambda \geq 0$ *the problem (1) has* $2\lambda + 1$*- parametric set of solutions;*

for $\lambda < 0$ *the problem (1) either has no solution or admits only one solution provided that* $f(z)$ *satisfies* $2|\lambda| - 1$ *real relations.*

A particular case as $f(z)$ does not satisfy the solvability relations and the problem (1) does not have a solution is presented in [1].

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Analysis of Hamiltonian dynamics in optimal control problems of optimization of resource productivity

In the paper, the optimal control problem is considered to maximize the utility function of the infinite time horizon

$$J(x_1(\cdot), x_2(\cdot), u(\cdot)) = \int\limits_0^{+\infty} e^{-\rho t} \left(\ln x_1(t) - \ln x_2(t) + \ln\left(1 - u(t) - x_2(t)\right)\right) dt, \quad u(t) \in [0, \overline{u}]$$

over control processes $\big(x_1(t), x_2(t), u(t)\big)$ of the dynamic system

$$\dot{x}_1(t) = x_1(t)\,(x_1(t) - A_1 u(t) + B)\,, \quad \dot{x}_2(t) = x_2(t)\,(x_1(t) - A_2 u(t))\,, \quad x_i(0) = x_i^0,\, i = 1, 2$$

satisfying the initial conditions and subject to constraints for the control parameter $u(t)$.

This problem is based on the model of optimization of the resource productivity by controlling investments $u(t)$ [2]. The problem is investigated within the Pontryagin maximum principle [1]. The domain of existence of steady states is described in terms of the model parameters. If the steady state has the saddle character then one can construct the nonlinear stabilizer. The stabilizer allows searching optimal solutions on the basis of stabilized trajectories which are close to optimal solutions and inherit their asymptotic properties in a vicinity of the steady state [3].

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Lie Spheres, Spherical Monogenics, and Small Dimensions

This talk will give an overview of the concepts of spherical monogenics on the Lie sphere, as described in the works of F. Sommen [1], and analyze its restriction to smaller dimensions. This line of thought was inspired by the earlier work of M. Morimoto [2].

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A boundary value problem for monogenic functions

The 2^n real-valued components of a monogenic function in $\mathbf{R^{n+1}}$ are solutions of the Laplace equation. However they are not independent of each other because they are connected by the Cauchy-Riemann system in $\mathbf{R^{n+1}}$ consisting of 2^n (real) first order equations. So, we can not arbitrarily prescribe the boundary values of all of the real-valued components. Taking into account this fact, we will show the solution of a Dirichlet boundary value problem in $\mathbb{R}^3$ for monogenic functions in a cylindrical domain. The underlying Clifford algebra depends on parameters in case the usual structure relations $e_j^2 = -1$ and $e_i e_j + e_j e_i = 0$ are replaced by the more general ones $e_j^2 = -\alpha_j$ and $e_i e_j + e_j e_i = 2\gamma_{ij}$. If the parameters of the Clifford algebra depend on the variable $x \in \mathbb{R}^3$, then the coefficients of the Cauchy-Riemann system for monogenic functions depend on x. We also show boundary value problems for the inhomogeneous generalized Cauchy-Riemann equation.

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Commutative C^*-algebras of Toeplitz operators on the Bergman space

We give a complete characterization of all commutative C^*-algebras generated by Toeplitz operators acting on the weighted Bergman spaces over the unit disk. Each such algebra is generated by Toeplitz operators whose bounded measurable symbols are invariant under the action of a maximal abelian subgroup of Moebius transformation of the unit disk, and is isomorphic to the algebra of sequences (or functions) that are slowly oscillate in a certain specific sense.

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Discrete singular integrals in a half-space

We consider Calderon – Zygmund singular integral [1] in the discrete half-space $\mathbb{Z}^m_{h,+}$, where $\mathbb{Z}^m_h$ is entire lattice (modulo $h, h > 0$) in $\mathbb{R}^m$, and prove, that the discrete singular integral operator is invertible in $L_2(\mathbb{Z}^m_{h,+})$ iff such is its continual analogue [2]. The key point for this consideration takes solvability theory of so-called periodic Riemann boundary problem, which is constructed by authors.

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Minimization of Hartree type functionals

The talk treats solitary waves associated with the Hartree type functional in external Coulomb potential. The functional associated with this problem is (see [1]) [1])

$$\mathcal{E}(u) = \frac{1}{2}\|\nabla u\|^2_{L^2} + \frac{1}{4}A(u^2) - \frac{E^2}{2}\int_{\mathbf{R}^3} \frac{|u(x)|^2}{|x|}\, dx. \tag{1}$$

with fixed L^2- norm

$$\int_{\mathbf{R}^3} |u(x)|^2 dx = E^2. \tag{2}$$

Here and below

$$A(f) = \int_{\mathbf{R}^3}\int_{\mathbf{R}^3} \frac{f(x)f(y)}{|x-y|} dydx. \tag{3}$$

The positive radial minimizers have to satisfy the ordinary differential equation

$$-ru''(r) - 2\,u'(r) + r\int_r^\infty \left(\frac{1}{s} - \frac{1}{r}\right) u^2(s)s^2 ds\; u(r) + ru(r) = 0 \tag{4}$$

together with the constraint

$$\int_0^\infty u^2(s)s^2 ds = E^2. \tag{5}$$

We study the expansion for external domains of type $r \geq 1$ by the aid of the ansatz

$$u(r) = \frac{e^{-r}}{r} v(r), \tag{6}$$

where is $v(r)$ defined by

$$v(r) = v(r,b) = b\left(1 + \sum_{k=1}^{\infty} (-1)^k b^{2k} \omega_k(r)\right). \tag{7}$$

We shall see that suitable recurrence relation for the coefficient can be used in order to establish uniform bound of type

$$0 < \omega_k(r) < \frac{C^k e^{-2rk}}{r^{2k}k} \tag{8}$$

and guarantee the uniform convergence of the series (7).

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Topological properties of regular generalized function algebras

We investigate the topological density of various subalgebras of regular generalized functions in the Colombeau algebra $\mathcal{G}(\Omega)$ of generalized functions with its natural (so-called sharp) topology. In contrast with the algebra of smooth functions which is a dense subset of the space of $\mathcal{C}^\infty$-smooth functions, we show that the subalgebra $\mathcal{G}^\infty(\Omega)$ is not dense in the algebra $\mathcal{G}(\Omega)$.

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Smoothing Operators, Colombeau Algebras and Generalised Differential Geometry

From the point of view of applications the great advantage of using Colombeau algebras is the way in which they represent distributions as families of smooth functions. In this talk we will examine the relationship between smoothing operators and Colombeau algebras based on some recent work of Nigsch [1]. We show that in the case of distributional tensor fields this leads to a Colombeau algebra of generalised tensor fields which has all the ingredients necessary to develop a natural theory of distributional differential geometry.

For the case of scalar fields a smoothing is simply a linear map from distributions to the space of smooth functions. By a variant of the Schwartz kernel theorem these correspond precisely to the space of smoothing kernels. This leads to a natural description of a (large) basic space $\mathcal{E}(M)$ for the corresponding Colombeau algebra together with canonical embeddings of both distributions and smooth functions. This results in a version of the algebra given in [2] in which the basic space is enlarged to allow position dependant smoothing kernels. For the case of distributional tensor fields on a manifold a similar analysis shows that the space of smoothing operators is topologically isomorphic to the tensor product of transport operators with smoothing kernels

$$\mathcal{L}_b(\mathcal{D}'^r_s(M), \mathcal{T}^r_s(M)) \cong \mathrm{TO(M,M)} \otimes \mathrm{C}^\infty(\mathrm{M}, \Omega^\mathrm{n}_\mathrm{c}(\mathrm{M})).$$

As with the scalar case this leads to a natural description of a basic space $\mathcal{E}^r_s(M)$ for the corresponding Colombeau algebra of generalised tensor fields together with canonical embeddings of both distributional and smooth tensor fields. From this point of view the transport operators used in [3] are a natural consequence of any smoothing of distributional tensor fields.
In the final part we will describe some applications to generalised differential geometry. We introduce the concept of distributional metric and show that it may be used to define a generalised connection and generalised curvature. The Geroch-Traschen class of metrics [4] is the largest class for which one can define a distributional curvature using conventional (linear) distributions. We show that for this class the generalised curvature of the embedded (Colombeau) metric is associated to the distributional curvature. However we also give examples of metrics outside this class for which the generalised curvature is associated to a distributional tensor field and give some preliminary results about the characterisation of such metrics.

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Time-dependent Schrödinger equation on cylinders and the n-torus

In this paper we study the solutions to the Schrödinger equation on some conformally flat cylinders and on the n-torus. First we apply an appropriate regularization procedure. Using the Clifford algebra calculus with an appropriate Witt basis, the solutions can be expressed as multi-periodic eigensolutions to the regularized parabolic-type Dirac operator. We study their fundamental properties, give representation formulas of all these solutions in terms of multiperiodic generalizations of the elliptic functions in the context of the regularized parabolic-type Dirac operator.

Furthermore, we also develop some integral representation formulas. In particular we set up a Green type integral formula for the solutions to the homogeneous regularized Schrödinger equation on cylinders and $n-$tori. Then we treat the inhomogeneous Schrödinger equation with prescribed boundary conditions in Lipschitz domains on these manifolds. We present an L_p-decomposition where one of the components is the kernel of the first order differential operator that factorizes the cylindrical (resp. toroidal) Schrödinger operator. Finally, we study the behavior of our results in the limit case where the regularization parameter tends to zero.

This is a joint work with R.S. Kraußhar (Technische Universität Darmstadt).

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General Stieltjes moment problems for rapidly decreasing smooth functions

The problem of moments, as its generalizations, is an important mathematical problem which has attracted much attention for more than a century. It was first raised and solved by Stieltjes for positive measures. Boas and Pólya, independently, showed later that given an arbitrary sequence $\{a_n\}_{n=0}^{\infty}$ there is always a function of bounded variation F such that

$$a_n = \int_0^{\infty} x^n \mathrm{d}F(x)\,, \quad n \in \mathbb{N}\,. \tag{1}$$

A major improvement to this result was achieved by Durán, who was able to show the existence of regular solutions to (1). He proved in [1] that every Stieltjes moment problem

$$a_n = \int_0^{\infty} x^n \phi(x)\mathrm{d}x\,, \quad n \in \mathbb{N}\,, \tag{2}$$

admits a solution $\phi \in \mathcal{S}(0,\infty)$, that is, a solution in the Schwartz class of rapidly decreasing smooth functions with $\operatorname{supp}\phi \subseteq [0,\infty)$.

In this talk we discuss a result which significantly improves Durán's theorem quoted above. We shall replace the sequence of monomials in (2) by a rather general sequence of distributions $\{f_n\}_{n=0}^{\infty}$ with $\operatorname{supp} f_n \subseteq [0,\infty)$ and present conditions which ensure that every generalized Stieltjes moment problem

$$a_n = \langle f_n, \phi \rangle\,, \quad n \in \mathbb{N}\,,$$

has a solution $\phi \in \mathcal{S}(0,\infty)$.

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THE GENERALIZED LAPLACE, STIELTJES AND POTENTIAL INTEGRAL TRANSFORMS

There are many complicated problems whose solutions cannot be represented by the classical transforms but may be solved by integral transforms with various special functions as kernels.

Let us consider the generalization of the classical integral by help of the Wright hypergeometric function

$${}_p\Psi_q(z) \equiv_p \Psi_q\left[\begin{array}{c|c} (a_i;\alpha_i)_{1,p} & \multirow{2}{*}{z} \\ (b_j;\beta_j)_{1,q} & \end{array}\right],$$

where $z \in C; a_i, b_j \in C; \alpha_i, \beta_j \in R = (-\infty,+\infty); (\alpha,\beta) \neq 0; i = 1,2,...,p; j = 1,2,...,q.$

We study the following generalized integral transforms:

$$\tilde{L}\{f(x);y\} = \int_0^{\infty} e^{-xy}\; {}_1\Phi_1^{\tau,\beta}(a;c;-b(xy)^y) f(x)dx, \tag{1}$$

$$\tilde{L}_m\{f(x);y\} = \int_0^{\infty} x^{m-1} e^{-x^m y^m}\; {}_1\Phi_1^{\tau,\beta}(a;c;-b(x^m y^m)^y) f(x)dx, \tag{2}$$

here ${}_1\Phi_1^{\tau,\beta}(a;c;z)$ is the generalized confluent hypergeometric function [1]:

$$(3) \qquad {}_1\Phi_1^{\tau,\beta}(a;c;z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)} \int_0^1 t^{a-1}(1-t)^{c-a-1} \, {}_1\Psi_1\left[\begin{array}{c} (c,\tau); \\ (c,\beta) \end{array} \middle| \, zt^{\tau} \right] dt,$$

where $Rec > Rea > 0; \{\tau, \beta\} \subset R, \ \tau > 0, \beta > 0; \tau - \beta < 1; \Gamma(a)$ is the classical Γ- function. An in (2) $m = 1, b = 0$, we have the classical Laplace integral transform.

The generalized potential transform we take is of the following form:

$$(4) \qquad \tilde{P}_{m,1}\{f(x);y\} = \frac{\Gamma(c)}{\Gamma(a)} \int_0^{\infty} \frac{x^{m-1}}{x^m + y^m} \, {}_2\Psi_1\left[\begin{array}{c} (a,\tau);(1,\gamma) \\ (c,\beta) \end{array} \middle| -b\left(\frac{x^m}{x^m+y^m}\right)^{\gamma} \right] f(x)dx.$$

If in (4) $m = 2, b = 0$, then we have the classical potential integral transform:

$$(5) \qquad P\{f(x);y\} = \int_0^{\infty} \frac{x}{x^2+y^2} \, f(x)dx.$$

The generalized Stieltjes integral we consider is of the form:

$$(6) \qquad \tilde{G}_p\{f(x);y\} = \frac{\Gamma(c)}{\Gamma(a)\Gamma(p)} \int_0^{\infty} \frac{f(x)}{(x+y)^p} \, {}_2\Psi_1\left[\begin{array}{c} (a,\tau);(p,\gamma) \\ (c,\beta) \end{array} \middle| -b\left(\frac{x}{x+y}\right)^{\gamma} \right] dx.$$

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Finite Element Modelling and Statistical Analysis for Applanation Tonometry after Refractive Surgery

The aim of the present work is to study the effect of corneal changes after refractive surgeries on the reliability of applanation tonometry.

Two different surgery techniques (PRK and LASIK) and two types of applanation tonometers are discussed. PRK uses an excimer laser to ablate directly the outer surface of the cornea, while LASIK is a technique in which the inner layer of the cornea (stroma) is ablated. Maklakoff tonometer (MAT) measures the IOP by the area flattened by a prescribed force, Goldmann tonometer (GAT) assesses the IOP by the force required to flatten the prescribed area of the cornea.

Two-dimensional axisymmetric finite element model of the corneoscleral shell was constructed. Simulated corneoscleral shell was considered as two joined transversal isotropic shells with different mechanical properties. The corneal thickness was divided into four layers with different geometrical and elastic parameters. Transversal isotropy was assumed for each corneal layer: the elastic moduli in the meridional and circumferential directions are equal to each other and are up to 100 times greater than Young's modulus in the thickness direction. Nonlinear analysis was carried out using engineering simulation software ANSYS, Inc.

Statistical analysis was performed on 50 eyes undergoing PRK for myopia. The matched group treated with LASIK was chosen so that the average central corneal thickness and the average IOP before the surgery were close to each other in both subgroups. We analyzed the IOP measurements before and 3 months after the surgery.

Our simulation results are in a good agreement with the experimental data. After LASIK surgery the additional layer appears. The increasing the layers number makes stiffness of the cornea smaller, and therefore, makes smaller the IOP–values measured using both GAT and MAT. The results obtained by GAT are significantly more sensitive to all parameters of refractive surgery than those found with MAT for 10 g load.

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Numerical solutions of the inverse problem of pharmacokinetics

What happens to the drug in the body can be visualized by considering the body as being made up of a large number of compartments, each of which has a well-defined volume in which the drug is well mixed. In all we can visualize this whole process as a dynamic system describe by a system of ordinary differential equations of the form:

$$C'(t) = AC(t), \tag{1}$$

where $C(t)$ is a vector of concentrations in the different compartments. The coefficient matrix A describes how different compartments are connected. Coefficients in A can be constants as well as functions of concentrations.

Different types of nonlinear compartment models are covered in this report. Linear models are also considered: three-compartment model with elimination from central compartment and two-compartment model with absorption.

In real life situation we determine a series of time points at which blood samples are taken and plasma concentrations are measured.

Here inverse problem arises: it is required to find rate constants knowing concentration of a drug in the central compartment at the given moments of a time. So, we have a nonlinear system of M equations (M is the quantity of the measurement data points). The inverse problem can be written in the following operator form:

$$A(q) = f, \tag{2}$$

where

$A\colon \mathbb{R}^k \longrightarrow \mathbb{R}^M_+$ – nonlinear operator (coefficient k indicates a number of required constants and depends on the type of considered model),

$f = (f_1, f_2, \ldots, f_M)^T, \quad f_j = C_j,\ j = \overline{1, M}$ – measured concentration of a drug at the give moments of a time.

Inverse problem is solved by different iterative methods. The question of choosing initial approximations is covered in this report. It is shown that physical properties of initial approximations strongly affect on obtained solutions. The results of computational experiments are presented. It is demonstrated that the resolving ability of the inverse problem can be improved by varying of the location of measurement data points.

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Integral Representations in Elliptic Complex Numbers

Generalized Complex Numbers are defined as complex numbers of the form $z = x + iy$ where the product of two complex numbers is induced by the relation $i^2 = -\beta i - \alpha$, for α and β real numbers subjected to the ellipticity condition $4\alpha - \beta^2 > 0$. Since the generalized complex numbers are isomorphic to the ordinary complex numbers, then from the point of view of the algebra they are structurally equal. In contrast to this fact, in Analysis a significant gain is obtained when we consider the Cauchy-Riemann operator acting on a complex valued function $f(z) = u + iv$ in the generalized complex algebra and considering functions in the kernel of this operator as the holomorphic ones. As there exist differentiable functions that are not holomorphic in the ordinary sense, yet they are holomorphic for some suitable choice of real numbers α and β, a more encompassing concept of holomorphicity is obtained.

The aim of this talk is to give a generalization of some integral formula for functions valued in the elliptic complex algebra. In particular a Cauchy integral representation formula is obtained for a generalized class of holomorphic functions.

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Irregular Samplings in Hardy Spaces and Reproducing Kernel Hilbert Space

We prove a new sampling formula for functions in the Hardy space in the unit disc using reproducing kernel Hilbert space techniques

Theorem. *Let f be a Hardy function in the unit disc, and $\{F_m\}_0^\infty$ be its Maclaurin coefficients. Let $\{z_k\}_{k=1}^\infty$ be arbitrary distinct complex points on $\mathbb{D}$, convergent to $z_0 \in \mathbb{D}$,*

$$\tilde{a}_{lk}^n = \frac{\prod_{j=1}^{n} [(1 - z_k\overline{z}_j)(1 - \overline{z}_l z_j)]}{(1 - z_k\overline{z}_l) \prod_{j=1,j\neq k}^{n} (z_k - z_j) \prod_{j=1,j\neq l}^{n} (\overline{z}_l - \overline{z}_j)}.$$

Then

$$\lim_{n\to\infty} \sum_{k,l=1}^{n} f(z_k)\tilde{a}_{lk}^n \frac{1}{1 - z\overline{z}_l} = f(z), \quad z \in \mathbb{D},$$

and

$$F_m = \lim_{n\to\infty} \sum_{k,l=1}^{n} f(z_k)\, \tilde{a}_{lk}^n\, \overline{z}_l^m, \quad m \geq 0.$$

Sampling formulas for Hardy functions in the right-half plane are also derived. As a consequence, a new inverse formula for the Laplace transform requiring data on a bounded interval is obtained

Theorem. *Let $\{p_k\}_{k=1}^\infty$ be a sequence of distinct numbers on the right half plane $\mathbb{C}_+$, that is either convergent to $p \in \mathbb{C}_+$, or $p_k = \alpha k + \beta$ for some $\alpha, \beta > 0$, and any $k > 0$. Then the Laplace inverse $f(t)$ of $F(z)$ can be determined from $\{F(p_k)\}_{k=1}^\infty$ by the formula*

$$f(t) = \lim_{n\to\infty} \sum_{k,l=1}^{n} F(p_k) \frac{\prod_{j=1}^{n} \left[(p_k + \overline{p}_j)(\overline{p}_l + p_j)\right]}{(p_k + \overline{p}_l) \prod_{j=1,j\neq k}^{n} (p_k - p_j) \prod_{j=1,j\neq l}^{n} (\overline{p}_l - \overline{p}_j)} e^{-\overline{p}_l t},$$

where the convergence is in $L^2(\mathbb{R}_+)$ norm.

Upper bounds for the truncation errors for some sampling formulas are also provided.

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The equality of the homogeneous and the Gabor wave front set

We prove that Hörmander's global wave front set and Nakamura's homogeneous wave front set of a tempered distribution coincide. In addition we construct a tempered distribution with a given wave front set, and we develop a pseudodifferential calculus adapted to Nakamura's homogeneous wave front set. This is joint work with René Schulz.

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Singularities of solutions to the Cauchy problem for a class of second-order hyperbolic operators

We deal with the Cauchy problem for second-order hyperbolic operators in the case where the coefficients of their principal parts are real analytic functions depending only on the time variable. We shall give outer estimates of the wave front sets of solutions to the Cauchy problem under the conditions given in [1]. Under these conditions the Cauchy problem is C^∞ well-posed, and the conditions are necessary for C^∞ well-posedness when the space dimension is less than 3 or the coefficients of the principal parts are semi-algebraic functions (*e.g.*, polynomials). More precisely, we shall show that the wave front sets of solutions are included in the union of broken null bicharacteristics emanating from the singularities of data in non-trivial cases.

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ON RIEMANN PROBLEMS FOR SINGLE-PERIODIC POLYANALYTIC FUNCTIONS

In this article, Riemann-type boundary-value problem of single-periodic polyanalytic functions has been investigated. By the decomposition of single-periodic polyanalytic functions, the problem is transformed into n equivalent and independent Riemann boundary-value problems of single-periodic analytic functions. Finally, we obtain the explicit expression of the solution and the conditions of solvability for the single-periodic polyanalytic functions.

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Effective Damping Mechanisms in Mathematical Aeroelasticity

We analyze a classical nonlinear coupled system of PDEs known as a *flow-plate interaction* which models the dynamics of a thin plate immersed in an inviscid potential flow [1, 4, 9]. The flow equation is given by a “perturbed” wave equation, while the plate is modeled by an irrotational von Karman or Berger plate; the dynamics are strongly coupled.

We address various mechanical dissipation mechanisms and their effectiveness in the context of long-time dynamics for the flow plate system. Specifically, we are concerned with global attracting sets [1] for the dynamics of the plate in the presence of a (naturally occuring or imposed) damping mechanism. We consider various configurations of the flow-plate interaction, and present results in the case of (a) fully supported interior damping (provided by the flow in certain configurations) [3], (b) geometrically constrained interior damping (whose support is restricted to a collar of the boundary) [5, 6], and (c) nonlinear boundary damping acting through a *hinged* boundary condition [7, 8]. We discuss the current results about the existence and qualitative properties of attracting sets, and the strong convergence of full flow-plate trajectories to *stationary points* [1, 2]; time permitting, we will discuss open problems for the cases of boundary damping and localized interior damping.

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Diffusion phenomena for partially dissipative hyperbolic systems

In this talk we consider hyperbolic systems with a partial dissipation affecting only some of the modes directly. We show that under suitable coupling properties (in form a Kalman rank condition imposed on the coefficient matrices) solutions have a diffusive structure und provide decay estimates for them as well as an adapted diffusion phenomenon. The result applies among others to the linearised damped Timoshenko system.

The talk is based on results from [2] with full proofs appearing in [1].

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Preprint, arXiv:1110.0797

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The principal symbol map for paired Lagrangian distributions

Paired Lagrangian distribtions were introduced by Melrose-Uhlmann [3] in order to achieve a symbolic parametrix construction for operators of real-principal type. An improved version was later given by Laubin-Willems [2] which relies on ideas from Bony's 2-microlocal calculus [1]. In this talk, we introduce a classical calculus for paired Lagrangian distributions, define the principal symbol map, and study certain cases of compositions when the kernels of the operators involved belong to this class.

Based on joint work with Nguyen Nhu Thang [4-6].

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Degenerate pseudodifferential operators of Vishik-Grushin type

We develop a symbol calculus for a class of pseudodifferential operators that degenerate in a specific way along a regular submanifold. Differential operators L in this class are of the form

$$L = \sum_{|\alpha|+|\beta|\leq m} \sum_{|\gamma|\geq|\alpha|+|\beta|(l_*+1)-p} a_{\alpha\beta\gamma}(x,y)\, x^\gamma D_x^\alpha D_y^\beta,$$

in local coordinates $(x,y) \in \mathbb{R}^d \times \mathbb{R}^q$ near $x = 0$, where $a_{\alpha\beta\gamma} \in C^\infty(\mathbb{R}^d \times \mathbb{R}^q)$ and the degeneracy occurs at $x = 0$. Here, $l_* \in \mathbb{Q}_+$ describes the kind of degeneracy under study, $m \in \mathbb{N}_0$, $p \in \mathbb{Z}$, and $p \leq m$. For instance, one has $d = 1$, $l_* = 1/2$, $m = p = 2$ for the Tricomi operator $\partial_x^2 + x\Delta_y$.

As an application, well-posedness of a certain class of boundary-value problems for PDEs of mixed type, where the hyperbolic region is sandwiched between elliptic regions, is proved.

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Flow in canal with rough bottom

To solve the problem we propose to replace the rough bottom (RB) by a smooth one in two ways. 1. The RB can be replaced by a layer of different viscous fluid, with viscosity changed according to Einsteinian theory of emulsion. 2. The RB can be treated as porous medium, after the homogenization theory, [1]-[5].

For illustration a two-dimensional but one-directional flow after the plane trough with rough bottom is considered and the exact results of analytical calculations are presented for the both ways of approximation. On the interface the continuity of velocity and shear stress is postulated. If the thickness of RB is fraction of the total fluid height the epsilon order then correction to fluid velocity distribution is also of the epsilon order.

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Hilbert–Schmidt and Trace Class Pseudo-Differential Operators for Weyl Transforms

Pseudo-differential operators with operator-valued symbols for Weyl transforms are introduced. Conditions for these pseudo-differential operators to be Hilbert–Schmidt and in the trace class are given. A trace formula for these trace class pseudo-differential operators is also presented.

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Applications of Orlicz spaces in the theory of PDE: non-Newtonian fluids and abstract problems

We are interested in the existence of solutions to strongly nonlinear partial differential equations. We concentrate mainly on problems which come from dynamics of non-Newtonian fluids of a nonstandard rheology and abstract theory of elliptic and parabolic equations. In considered problems the nonlinear highest order term is monotone and its behaviour – coercivity/growth condition – is given with help of some general convex function. In our research we would like to cover both cases: sub- and super-linear growth of nonlinearity. Such a formulation requires a general framework for the function space setting, therefore we work with non-reflexive and non-separable Orlicz and Musielak-Orlicz spaces. Within the presentation we would like to emphasise problems we have met during our studies, their reasons and methods which allow us to achieve existence results.

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Exact Meromorphic Stationary Solutions of the Cubic-Quintic Swift-Hohenberg Equation

In this presentation, based on Nevanlinna theory and Painlevé analysis [1, 2], we will first of all show that all the meromorphic stationary solutions of the cubic-quintic Swift-Hohenberg equation, which models many circumstances of physical systems, belong to the class W (like Weierstrass)[1], consisting of elliptic functions and their degenerations, i.e., elliptic functions, rational functions of one exponential $\exp(kz), k \in \mathbb{C}$ and rational functions of z. Then we obtain them all explicitly by the method introduced in [3], and some of them appear to be new solutions.

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The Stochastic Wave Equation with an Interval Valued Parameter

We consider the solutions u_c of the stochastic wave equation in three space dimensions

$$\partial_t^2 u_c - c^2 \Delta u_c = \dot{W} \qquad\qquad u_c : \Omega \to \mathcal{S}'\left(\mathbb{R}^4\right)$$

denoting by $\dot{W}$ the white noise with support in $[0,\infty)\times\mathbb{R}^3$. It is a generalized stochastic process on a probability space (Ω, Σ, μ). A suitable choice for Ω is the space of tempered distributions $\mathcal{S}'(D)$.

Modelling the parameter c as an interval means to investigate the function

$$X : \Omega \to P\left(\mathcal{S}'\left(\mathbb{R}^4\right)\right)$$
$$\omega \mapsto \{u_c(\omega), c_1 \leq c \leq c_2\}$$

In this contribution we show that X fulfils the Borel measurability condition

$$X^-(B) := \{\omega \in \Omega : X(\omega) \cap B \neq \emptyset\} \in \mathcal{B}(\Omega) \qquad\qquad \forall B \in \mathcal{B}\left(\mathcal{S}'\left(\mathbb{R}^4\right)\right)$$

and therefore is a random set in the general sense of Molchanov [1].

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The hyperbolic equations in the curved spacetime

The talk is concerned with the global in time solutions of the Cauchy problem for matter waves propagating in the curved spacetimes, which can be, in particular, modeled by cosmological models. We examine the global in time solutions of some class of semililear hyperbolic equations, such as the Klein-Gordon equation, which includes the Higgs boson equation in the de Sitter spacetime and Einstein & de Sitter spacetime. In particular we show that the Klein-Gordon equation in the de Sitter spacetime obeys the Huygens' principle only if the physical mass m of the scalar field and the dimension n of the spatial variable are tied by the equation $m^2 = (n^2 - 1)/4$. Moreover, we define the incomplete Huygens' principle, which is the Huygens' principle restricted to the vanishing second initial datum, and then reveal that the massless scalar field in the de Sitter spacetime obeys the incomplete Huygens' principle and does not obey the Huygens' principle, for the dimensions $n = 1, 3$, only. Thus, in the de Sitter spacetime the existence of two different scalar fields (in fact, with $m = 0$ and $m^2 = (n^2 - 1)/4$), which obey incomplete Huygens' principle, is equivalent to the condition $n = 3$. For $n = 3$ these two values of the mass are the endpoints of the so-called in quantum field theory the Higuchi bound.

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Biomechanical model of the human eye on the base of nonlinear shell theory

The goal of this work is the development of a biomechanical model of the human eye and to prove software simulation systems of measuring the intraocular pressure (IOP) by an optical analyzer. We numerically simulate the eye deformation when the IOP is measured using the Ocular Response Analyzer developed by the USA company Reichert. The biomechanical model includes a cornea and a sclera, which are considered as axisymmetrically deformable shells of revolution with fixed boundaries; the space between these shells is filled with incompressible fluid. Nonlinear shell theory is used to describe the stressed and strained state of the cornea and sclera. The optical system is calculated from the viewpoint of the geometrical optics. Dependences between the pressure in the air jet and the area of the surface reflecting the light into a photo detector for the different thickness of the cornea were obtained. The shapes of the regions on the cornea surface were found from which the reflected light falls on the photo detector. First, the light is reflected from the center of the cornea, but then, as the cornea deforms, the light is reflected from its periphery. The numerical results make it possible to better interpret the measurement data. This work was supported by a grant 13-01-00801 from the Russian Foundation for Basic Research.

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Spectral Analysis of Daubechies Operator

By using the theory of Fourier ultra hyperfunctions, we clarified the complex analytic properties of eigenvalues of Daubechies Operator. Especially we can recover symbol function of Daubechies Operator from its eigenvalues.

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Growth order of meromorphic solutions of some algebraic differential equations and Barsegains initial method

In this talk, we introduce some results on estimate the growth order of meromorphic solutions of some algebraic differential equations by using Barsegian's initial method [1], we estimate the growth order of entire or meromorphic solutions of some algebraic differential equations with rational or transcendental meromorphic function coefficients[5, 6]. Our results improve or extend the related results of Barsegian et al.[3], Bank [2], Li and Feng[4]. Moreover, we answer positively a question in [4], and also give some examples to show that our results are sharp in some special cases.

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Necessary and Sufficient Conditions for Associated Differential Operators in Quaternionic Analysis and Applications to Initial Value Problems

This paper deals with the initial value problem of type

$$\begin{aligned} \frac{\partial u}{\partial t} &= \mathcal{L}u := \sum_{i=0}^{3} A^{(i)}(t,x)\frac{\partial u}{\partial x_i} + B(t,x)u + C(t,x) \\ u(0,x) &= u_0(x) \end{aligned}$$

in the space of generalized regular functions in the sense of QUATERNIONIC ANALYSIS satisfying the differential equation

$$\mathcal{D}_\lambda u := \mathcal{D}u + \lambda u = 0,$$

where $t \in [0,T]$ is the time variable, x runs in a bounded and simply connected domain in $\mathbb{R}^4$, λ is a real number, and $\mathcal{D}$ is the CAUCHY-FUETER operator. We prove necessary and sufficient conditions on the coefficients of the operator $\mathcal{L}$ under which $\mathcal{L}$ is *associated* with the operator $\mathcal{D}_\lambda$, i.e. $\mathcal{L}$ transforms the set of all solutions of the differential equation $\mathcal{D}_\lambda u = 0$ into solutions of the same equation for fixedly chosen t. This criterion makes it possible to construct operators $\mathcal{L}$ for which the initial value problem is uniquely soluble for an arbitrary initial generalized regular function u_0 by the method of *associated spaces* constructed by W. Tutschke [1] and the solution is also generalized regular for each t.

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Viscoelastic model of the periodontal ligament

Periodont is a ligament which holds the tooth root in the bone alveoli and performs a number of functions. The main ones are the retention and shock absorbing function. Periodont is a complicated structure which properties still not completely known.
The periodontal ligament can be considered as a viscoelastic layer between the tooth root and bone when the tooth root is subjected to the load acting certain time. In particular, in [1] and [2], indicates the presence of the viscoelastic properties of the periodontal ligament. Once the load is zero and the stress relaxation occurs, surrounding the tooth root periodont will be restored. The stress-strength state of the periodontal ligament that occurs under the action on a tooth of constant orthodontic loads is defined taking into account their viscoelastic properties. It is assumed that the outer and inner surfaces of the periodontal membrane are determined by the equations of elliptic hyperboloid $F_0(x_1,\ x_2,\ x_3)$ and $F(x_1,\ x_2,\ x_3)$ respectively:

$$F_0(x_1,\ x_2,\ x_3) = F(x_1,\ x_2,\ x_3) + h_0 = 0,$$
$$F(x_1,\ x_2,\ x_3) = x_3 - \frac{H}{\sqrt{1+p^2}-p}\left(\sqrt{\frac{x_1{}^2}{a^2} + \frac{x_2{}^2}{b^2} + p^2} - p\right) = 0,$$

where H - height of tooth root; p - parameter characterizing the fillet of the tooth root top; a, b - ellipse axles in the tooth root section $z = H$.
The displacement vector is written as follows:

$$\vec{u} = \frac{1}{h_0}\left(F(x_1,\ x_2,\ x_3) + h_0\right)\left(\vec{u}^{(0)}(t) + \vec{\varphi}(t) \times \vec{r}\right)$$

here $\vec{u}^{(0)}(t)$ - time dependence of the translational displacements of the tooth root; $\vec{\varphi}(t)$ - time dependence of the rotation angles of the tooth root; $\vec{r}$ - the radius vector drawn from the origin. On the basis of motion equations and expressions for displacements, stresses and strains, using the Laplace transform, the vertical movement of the tooth root in the viscoelastic periodontal ligament under constant load over time is defined. The expressions for the displacement of the tooth root, depending on the time for the exponential relaxation kernels for Maxwell and Voigt models are obtained. Time dependence of the translational displacement of the premolar's root under action of vertical load are built.

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Stability of linear periodic Hamiltonian systems on time scales

In this talk, I will give a new stability criterion for planar periodic Hamiltonian systems improving the results in the literature. The method is based on an application of the Floquet theory recently established in [1] and the use of a new generalized zero concept introduced in [2]. The results obtained not only unify the related continuous and discrete ones but also provide sharper stability criteria for the discrete case.

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Generalized and fundamental solutions of Biot equations for moving loads

Here two-component medium of Biot [1] consisting of solid and fluid components is considered under action of moving loads. It is assumed that the mass forces move with c constant velocity along the axis z and represented as $G_i = G_i(x_1, x_2, z + ct)$ and Biot motion equations are

$$(\lambda + \mu)u_{sj},_{ji} + Qu_{fj},_{ji} + \mu u_{si},_{jj} + G_{si} = c^2(\rho_{11}u_{si},_{33} + \rho_{12}u_{fi},_{33})$$

$$Qu_{si},_{ji} + Ru_{fj},_{ji} + G_{fi} = c^2(\rho_{12}u_{si},_{33} + \rho_{22}u_{fi},_{33}), \quad (1)$$

where u_{si}, u_{fi} are the displacements of elastic and fluid components, G_{si}, G_{fi} are the body forces acting respectively on the each components, $\rho_{11} = (1-m)\rho_s - \rho_{12}$, $\rho_{22} = m\rho_f - \rho_{12}$, ρ_s and ρ_f are densities of the elastic component and fluid, m is the porosity of the medium. The elastic stress tensor components are $\sigma_{ij} = \mu(u_{si},_j + u_{fi},_j) + (\lambda u_{sk},_k + Qu_{fk},_k)\delta_{ij}$, $p = -(Qu_{sk},_k + Ru_{fk},_k)/m$ is the fluid pressure, λ, μ, Q, R are the mediums constants. For subsonic load: $c < \min\{c_1, c_2, c_3\}$ we have elliptic equations. For supersonic load: $c > \max\{c_1, c_2, c_3\}$ - strong hyperbolic equations.

For transonic loads: $c \in (c_1, c_2), c \neq c_3$ equations (1) - mixed hyperbolic-elliptic equations. Here c_1, c_2 describe the propagation velocity of longitudinal waves, the third c_3 - a shear wave $(c_2 < c_3 < c_1)$.

The fundamental and generalized solutions have been constructed for such velocities. The conditions on fronts of shock waves have been obtained:

$$[u_s]_F = [u_f]_F = 0,$$
$$[\mu u_{si},_j n_j + ((\lambda+\mu)u_{sj},_j + Qu_{fj},_j)n_i - c^2(\rho_{11}u_{si},_3 + \rho_{12}u_{fi},_3)n_3]_F = 0,$$
$$[(Qu_{sj},_j + Ru_{fj},_j)n_i - c^2(\rho_{12}u_{si},_3 + \rho_{22}u_{fi},_3)n_3]_F = 0,$$

n is the unit normal to the wave front.

Constructed fundamental solutions can be used for solving boundary-value problems in a Biot medium with cylindrical boundaries, under action of moving loads. A similar problem for an elastic medium is considered in [2]. Solutions obtained here can also be used to investigations the massif's dynamics in the neighborhood of underground constructions such as tunnels, transport pipelines depending on the properties of water saturation of the medium, the velocity and type of existing transport loads.

[1] Biot, M.A., *Mechanics of deformation and acoustic propagation in porous media*, J. Applied Physics, **33**, 1482-1498 (1962).

[2] Alexeyeva, L. A., *Singular border integral equations of the BVP of elastodynamics in the case of subsonic running loads*, Differential equations, **46**, (4), 512-519 (2010).

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Fixed point theorems for multivalued mappings

This talk is devoted to studying of some properties of multivalued mappings in Euclidean space. There were proved theorems on a fixed point for multivalued mappings whose restrictions to some subset in the closure of a domain of Euclidean space satisfy "an acute angle condition" or "a strict acute angle condition". Obtained results are still true also in the case of non continuous mappings.

Theorem. Let D be domain of Euclidean space $X = E^n$. Let $K \subset \overline{D}$ be a subset in the closure of this domain and let there exists a restriction F_1 of multivalued mapping $F : \overline{D} \to E^n = Y$ on subset K, which satisfied "a coacute angle condition" and convex hull conv $F_1(K)$ is compact. If $F(\overline{D}) \supset convF_1(K)$, then $0 \in F(\overline{D})$.

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Weak monotonicity in number series and Hardy inequalities

Two well-known tests for the convergence/divergence of an infinite series state

1) If a series

$$\sum_{k=1}^{\infty} a_k \tag{1}$$

converges, then its term a_k tends to 0;

2) If a convergent series (1) is monotone than $ka_k \to 0$.

In fact the assumption of monotonicity can be replaced by a much weaker condition called weak monotonicity (WM) (see [2]). A sequence $\{a_k\}$ is WM if

$$a_k \le Ca_n \quad \text{for any} \quad k \in [n, n+n]. \tag{2}$$

Let $m_{k+1} \ge m_k \ge 0$. What kind of sharp conditions should be imposed on $\{a_k\}$ to guarantee $m_k a_k \to 0$? For example what should be assumptions for $(\ln k)a_k \to 0$?

To get the required conditions we need simply to replace the last n in (2) with m_n $(\ln n)$. Thus we get a class $WM(\mathbf{m})$ of weak monotone sequences with respect to $\{m_n\}$.

It turns out that several convergence tests for number series such as Cauchy, Maclaurin and Schlömilch tests can be extended also for these classes of sequences. Moreover the class $WM = WM(\{n\})$ appears to be the limiting class in many cases.

We will also consider the Hardy inequalities

$$\sum_{n=1}^{\infty} n^{\beta}\Big(\frac{1}{n}\sum_{k=1}^{n} a_k\Big)^p \le C\sum_{n=1}^{\infty} a_n^p n^{\beta}, \qquad \beta < p-1,$$

and

$$\sum_{n=1}^{\infty} n^{\beta}\Big(\frac{1}{n}\sum_{k=n}^{\infty} a_k\Big)^p \le C\sum_{n=1}^{\infty} a_n^p n^{\beta}, \qquad \beta > p-1,$$

where $0 < p < 1$ for class of weak monotone sequences with respect to $\{m_n\}$ and observe that they do not hold for $\{a_n\} \in WM(\{m_n\})$, where $m_n = o(n)$, unlike the case $\{a_n\} \in WM$ (cf. [1]).

[1] Leindler, L., *Inequalities of Hardy-Littlewood type*, Anal. Math. **2** (2), 117-123 (1976).

[2] Liflyand, E., Tikhonov, S., Zeltser, M., *Extending tests for convergence of number series*, J. Math. Anal. Appl. **377** (1) , 194-206 (2011).

[3] Tikhonov, S. and Zeltser, M., *Weak monotonicity concept and its applications*, Trends in Mathematics, Springer Basel (to appear).

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Solution of the Cauchy problem for two-dimensional system of quasilinear equations of hyperbolic type

The algorithm of solving of the two-dimensional Cauchy problem for system of quasilinear equations of hyperbolic type proposed by the author. Using by conservation laws, we reduced this problem to linear integro-differential equation. This allowed not only to prove the existence of solutions, but also offer a numerical algorithm solving problems.

[1] Zhdanov, O. *Solving of mixed problem for a two-dimensional system of quasilinear hyperbolic partial differential equations*, Abstracts of 8th International ISAAC Congress, Moscow, 97 (2011).

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Choice(using by probabilistic inequalities) of the block diagram of reliability of an onboard complex of control of the small satellite

Various block diagrams of reliability of a control system of small spacecraft are considered in this paper. The technique of choice of the optimum scheme is developed, using by probabilistic inequalities. All possible options of reservation are considered and the theorem of the optimum scheme is proved.

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Constant scalar curvature Kähler metrics with cone singularities

This talk is about the Kähler cone metrics which are the Kähler metics with prescribed cone singularities. We will discuss the cone geodesics in the space of the Kähler cone metrics [1] and the constant scalar curvature Kähler cone metrics [2]. The Dirichlet problem of the geodesic equation in the space of Kähler cone metrics $\mathcal{H}_\beta$; that is equivalent to a homogeneous complex Monge-Ampère equation whose boundary values consist of Kähler metrics with cone singularities. We introduce a subspace $\mathcal{H}_C$ of $\mathcal{H}_\beta$ which we define by prescribing appropriate geometric conditions. Our main result is the existence, uniqueness and regularity of $C^{1,1}_\beta$ geodesics whose boundary values lie in $\mathcal{H}_C$. Moreover, we prove that such geodesic is the limit of a sequence of $C^{2,\alpha}_\beta$ approximate geodesics under the $C^{1,1}_\beta$-norm.

[1] Calamai, S. and Zheng, K., *Geodesics in the space of Kähler cone metrics*, arXiv:1205.0056 (2012).
[2] Zheng, K., *Constant scalar curvature Kähler metrics with cone singularities*, in preparation.

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Frame-based Efficient Computational Schemes for Discrete Time-frequency Analysis

Various time-frequency analysis techniques based on the Stockwell transform [1] characterize the temporal variation of a signal in a time-frequency domain with multi-scale analysis. They have been successfully used in many fields such as medicine, geophysics and oceanology. The continuous transforms provide graphically smooth time-frequency representations of a signal with details, while the transforms implemented using orthogonal bases [2] offer low computational cost but produce blocky time-frequency representations that may overlook subtle details. In this paper [3], we provide a frame-based computational framework that expand and unify various existing discrete Stockwell transforms. It not only provides flexibility to adjust the resolution of time and frequency, select the type of windows function, and tailor the ratio of redundancy , but also reserves the desirable mathematical properties of the conventional Stockwell transform such as absolutely-referenced phase and invertibility.

[1] Stockwell, R.G., Mansinha, L., Lowe, R.P., *Localization of the complex spectrum: the S-transform*, IEEE Transaction on Signal Processing, **44 (4)**, 998-1001 (1996).
[2] Stockwell, R.G., *A basis for efficient representation of the S-Transform*, Digital Signal Processsing, **17**, 371-393 (2007).
[3] Yan, Y.S., Zhu, H., *The generalization of the discrete Stockwell transforms*, the Proceedings of the 19th European Signal Processing Confernece, Barcelona, Spain (2011)

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Geometrical features of the soliton solution

It is well known, that integrable equations are solvable by the inverse scattering method [1]. Investigating of the integrable spin equations in (1+1), (2+1) dimensions are topical both from the mathematical and physical points of view. Integrable equations admit different kinds of physically interesting solutions as solitons, vortices, dromions etc. We consider an integrable spin M-I equation [2]. There is a corresponding Lax representation. And the equation allows an infinite number of integrals of motion. We construct a surface corresponding to soliton solution of the equation. Further, we investigate some geometrical features of the surface.

[1] Ablowitz M.J. and Clarkson P.A, *Solitons, Non-linear Evolution Equations and Inverse Scattering*, Cambridge University Press, Cambridge,1992.

[2] Myrzakulov R., Vijayalakshmi S., et all. *A (2+1)-dimensional integrable spin model: Geometrical and gauge equivalent counterparts, solitons and localized coherent structures*, J. Phys. Lett. A., **233A.**, 391-396 (1997).

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Nonlinear PDE as Immersions

Investigating of the nonlinear PDE including their geometric nature is one of the topical problems. With geometric point of view the nonlinear PDE are considered as immersions. We consider some aspects of the simplest soliton immersions in multidimensional space in Fokas-Gelfand's sense [1]. In (1+1)-dimensional case nonlinear PDE are given in the condition

$$A_t - B_x + [A, B] = 0, \qquad (1)$$

where A is 3×3 prescribed matrix, B is expressed by elements of the matrix A, $[A, B] = AB - BA$. Nonlinear PDE (1) are compatibility condition some system of linear equations [2]. In this case there is a surface with immersion function. We find the second quadratic form in Fokas-Gelfand's sense associated to one soliton solution of nonlinear Schrodinger equation.

[1] Ceyhan, O. Fokas, A.S., Gurses, M. *Deformations of surfaces associated with integrable Gauss-Mainardi-Codazzi equations*, J. Math. Phys., **41**, No.4, 2551-2270 (2000).

[2] Lakshmanan, M. Myrzakulov, R., et all.*Motion of curves and surfaces and nonlinear evolution equations in 2+1 - dimensions*, J. Math. Phys., **39**, No. 7, 3765-3771 (1998).

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Second-order periodic boundary value problem at resonance

We will discuss the existence of positive solutions for the following second-order periodic boundary value problem at resonance

$$(1)\qquad \begin{cases} x''(t) + h(t)x'(t) + f(t, x(t), x'(t)) = 0, \ t \in [0, T], \\ x(0) = x(T), \ x'(0) = x'(T), \end{cases}$$

where $f : [0, T] \times [0, \infty) \times \mathbb{R} \to \mathbb{R}$ and $h : [0, T] \to (0, \infty)$ are continuous functions.

Our method employs a Leggett-Williams norm-type theorem for coincidences due to O'Regan and Zima [1]. The talk is based on a joint paper [2].

[1] O'Regan, D. and Zima, M. *Leggett-Williams norm-type theorems for coincidences*, Arch. Math. **87**, 233–244 (2006).

[2] Zima, M. and Drygaś, P. *Existence of positive solutions for a kind of periodic boundary value problem at resonance*, Boundary Value Problems 2013, 2013:19.

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